SVOLGIMENTI ESONERO DI ANALISI 1
DEL 19/11/2025

$\boxed{\text{COMPITO A}}$

1) $$\frac{\sqrt{1+x+x^2}-1}{\sin(2x)} \underset{x\to 0}{\sim} \frac{x+x^2}{4x} = \frac{1+x}{4} \xrightarrow[x\to 0]{} \frac{1}{4}.$$

2) $$|x+1+iy| = x+iy-i(x-iy)$$
$$= x+iy-ix-y$$

Poiché $|w| \in \mathbb{R} \Rightarrow iy-ix = i(y-x) = 0$

$\Rightarrow x=y$. Ne segue che

$$\begin{cases} \sqrt{(x+1)^2+y^2} = x-y = 0 \\ y=x \end{cases}$$

$$\Rightarrow \begin{cases} (x+1)+x^2 = 0 \\ y=x \end{cases} \Rightarrow \begin{cases} x=-1 \\ x=0 \\ y=x \end{cases} \quad \text{IMPOSSIBILE}$$

3) $$a_n = \ln\left(\sqrt{\frac{1+n^{2\alpha}}{n^{2\alpha}}}\right) = \frac{1}{2}\ln\left(1+\frac{1}{n^{2\alpha}}\right) \xrightarrow[n\to\infty]{} 0$$

$\Longleftrightarrow 2\alpha > 0$ cioè $\alpha > 0$

Per $\alpha = 0$ $\quad a_n = \frac{1}{2}\ln 2 \xrightarrow[n\to\infty]{} \frac{1}{2}\ln 2.$

Per $\alpha < 0$ $\quad a_n \underset{n\to\infty}{\sim} \frac{1}{2} \ln(n^{-2\alpha}) \xrightarrow[n\to\infty]{} +\infty$

Per la convergenza di $\sum a_n$, possiamo considerare solo il caso $\alpha > 0$, perchè $a_n \xrightarrow[n\to\infty]{} 0$. Serie a termini positivi.

$$\ln\left(1+\frac{1}{n^{2\alpha}}\right) \sim \frac{1}{n^{2\alpha}}$$

$$\Rightarrow \sum a_n \approx \frac{1}{2} \sum \frac{1}{n^{2\alpha}} \begin{cases} \text{convergente se } 2\alpha > 1 \iff \alpha > \frac{1}{2} \\ \text{divergente se } \alpha \leq \frac{1}{2} \end{cases}$$

COMPITO B

1) $$\lim_{n\to+\infty} \frac{n^3+3}{n+1} = +\infty$$

$$\Rightarrow \lim_{n\to+\infty} \operatorname{arctg}\left(\frac{n^3+3}{n+1}\right) = \frac{\pi}{2}$$

$$\Rightarrow a_n \underset{n\to+\infty}{\sim} \frac{\pi}{2n^\alpha} \longrightarrow \begin{cases} 0 & \text{se } \alpha>0 \\ \frac{\pi}{2} & \text{se } \alpha=0 \\ +\infty & \text{se } \alpha<0 \end{cases}$$

$\sum a_n$ è a termini positivi.

$$\sum a_n \approx \frac{\pi}{2} \sum \frac{1}{n^\alpha} \begin{cases} \text{convergente se } \alpha>1 \\ \text{divergente se } \alpha\le 1. \end{cases}$$

2) $$\frac{e^{\operatorname{sen}(3x)}-1}{\operatorname{tg}\left[\ln(1+2x)\right]} \underset{x\to0}{\sim} \frac{\operatorname{sen}(3x)}{\ln(1+2x)}$$

$$\underset{x\to0}{\sim} \frac{3x}{2x} \xrightarrow[x\to0]{} \frac{3}{2}.$$

3) ⓐ $(iz^3+8)=0$ oppure ⓑ $z\bar{z}-\operatorname{Re}(z)-i=0$

Studiamo ⓐ:

$$z^3 = -\frac{8}{i} = 8i \Rightarrow z = 2\sqrt[3]{i}$$

$$z = 2\sqrt[3]{e^{i\frac{\pi}{2}}} \quad \Rightarrow \quad z_k = 2e^{\frac{i}{3}\left(\frac{\pi}{2}+2k\pi\right)}, \quad k=0,1,2$$

$$z_0 = 2e^{i\frac{\pi}{6}} = 2\left[\cos\left(\frac{\pi}{6}\right)+i\,\mathrm{sen}\left(\frac{\pi}{6}\right)\right]$$

$$= 2\left[\frac{\sqrt{3}}{2}+i\frac{1}{2}\right] = \sqrt{3}+i$$

$$z_1 = 2e^{i\frac{5}{6}\pi} = 2\left[\cos\left(\frac{5}{6}\pi\right)+i\,\mathrm{sen}\left(\frac{5}{6}\pi\right)\right]$$

$$= 2\left[-\frac{\sqrt{3}}{2}+i\frac{1}{2}\right] = -\sqrt{3}+i$$

$$z_2 = 2e^{i\left(\frac{9}{6}\pi\right)} = 2e^{i\frac{3}{2}\pi} = 2\left[\cos\left(\frac{3}{2}\pi\right)+i\,\mathrm{sen}\left(\frac{3}{2}\pi\right)\right]$$

$$= -2i$$

(b): $$|z|^2 - x - i = 0$$

$$\Rightarrow x^2+y^2-x-i=0$$

$$\begin{cases} x^2+y^2-x=0 \\ 1=0 \end{cases} \qquad \text{IMPOSSIBILE}$$

Abbiamo solo le tre soluzioni z_0, z_1, z_2.