SVOLGIMENTI PROVA SCRITTA DI ANALISI 1 del 20/3/2026.

1) $$a_n \sim \frac{\frac{6}{n^3}}{\left[\frac{1}{n}-\frac{2}{n^2}-\frac{1}{n}\right]} = -\frac{3}{n}$$

$\sum a_n \approx -3 \sum \frac{1}{n}$ divergente negativamente.

2) $D = \mathbb{R} - \{0\}$.

f è dispari: $f(-x) = \frac{e^{-(-x)^2/2}}{-x} = -\frac{e^{-x^2/2}}{x} = -f(x)$.

~~Intersez~~ Non ho intersezioni con gli assi.

$f(x) > 0 \iff x > 0$

$\lim_{x\to 0^\pm} f(x) = \frac{1}{0^\pm} = \pm\infty$ $\quad$ $x=0$ asintoto verticale da DX e SX.

$\lim_{x\to\pm\infty} f(x) = \frac{0}{\pm\infty} = 0^\pm$ $\quad$ $y=0$ asintoto orizzontale a $\pm\infty$.

$$f'(x) = e^{-\frac{x^2}{2}}\left(-\frac{1}{x^2}-1\right) = -\left(\frac{1}{x^2}+1\right)e^{-x^2/2} < 0 \quad \forall x\in D.$$

f decresce in $(-\infty,0)$ e in $(0,+\infty)$.

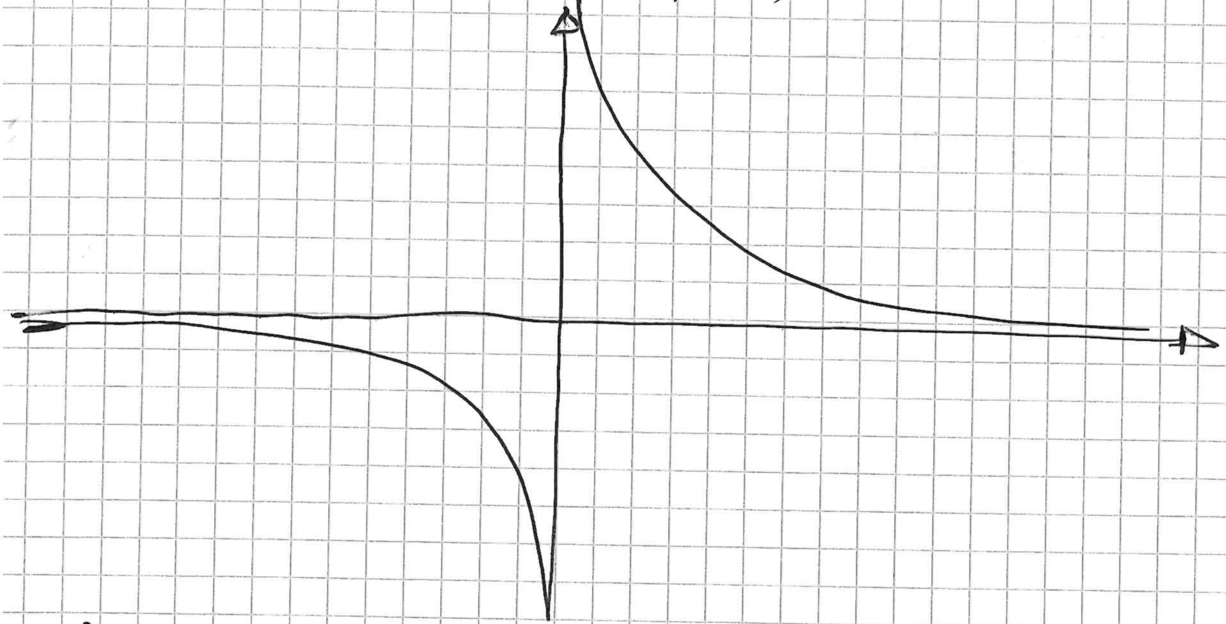
Non ammette MAX né MIN, relativi o assoluti.

$$f''(x) = -\left[-\frac{2}{x^3}+\left(\frac{1}{x^2}+1\right)(-x)\right]e^{-x^2/2}$$

$$= \frac{2+x^2+x^4}{x^3}e^{-x^2/2}$$

$x^4+x^2+2>0 \quad \forall x\in D. \quad \Rightarrow f''(x)>0 \iff x>0.$

f concava in $(-\infty, 0)$ e convessa in $(0, +\infty)$.



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3)
$$(1+i)z^3 = -8\sqrt{2}$$

$$z^3 = \frac{-8\sqrt{2}}{1+i} = \frac{-8\sqrt{2}}{2}(1-i) = 8\left[-\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right]$$

$$= 8\left[\cos\left(\frac{3}{4}\pi\right) + i\,\text{sen}\left(\frac{3}{4}\pi\right)\right]$$

$$z = 2\left[\cos\left(\frac{\frac{3}{4}\pi + 2k\pi}{3}\right) + i\,\text{sen}\left(\frac{\frac{3}{4}\pi + 2k\pi}{3}\right)\right]$$

$k = 0, 1, 2.$

$$z_0 = 2\left[\cos\left(\frac{\pi}{4}\right) + i\,\text{sen}\left(\frac{\pi}{4}\right)\right] = \sqrt{2}(1+i)$$

$$z_1 = 2\left[\cos\left(\frac{11}{12}\pi\right) + i\,\text{sen}\left(\frac{11}{12}\pi\right)\right]$$

$$z_2 = 2\left[\cos\left(\frac{19}{12}\pi\right) + i\,\text{sen}\left(\frac{19}{12}\pi\right)\right].$$

4) OMOGENEA ASSOCIATA:

$$\alpha^2-\alpha=\alpha(\alpha-1)=0 \qquad \Rightarrow \alpha_1=1;\ \alpha_2=0$$

$$\Rightarrow y_0(x)=C_1e^x+C_2$$

NON OMOGENEA: poiché $\alpha=0$ è radice del polinomio caratteristico, allora

$$y_P(x)=x(Ax^2+Bx+C).$$
$$y_P'(x)=3Ax^2+2Bx+C$$
$$y_P''(x)=6Ax+2B$$

$$\Rightarrow y_P''(x)-y_P'(x)=6Ax+2B-3Ax^2-2Bx-C=x^2$$

$$\begin{cases}-3A=1\\ 6A-2B=0\\ 2B-C=0\end{cases} \quad \begin{cases}A=-\frac{1}{3}\\ B=3A=-1\\ C=2B=-2\end{cases}$$

$$y_{NO}(x)=C_1e^x+C_2-\frac{1}{3}x^3-x^2-2x$$
$$y_{NO}'(x)=C_1e^x-x^2-2x-2$$

$$\begin{cases}y_{NO}(0)=C_1+C_2=0\\ y_{NO}'(0)=C_1-2=0\end{cases} \Rightarrow \begin{cases}C_1=2\\ C_2=-C_1=-2\end{cases}$$

$$\Rightarrow y(x)=2e^x-2-2x-x^2-\frac{1}{3}x^3$$

5) Integriamo per parti

$$\int_0^1 x^2 \sinh x\,dx = x^2 \cosh x \Big|_0^1 - \int_0^1 2x \cosh x\,dx$$

$$= \cosh 1 - 2\left[x \sinh x \Big|_0^1 - \int_0^1 \sinh x\,dx \right]$$

$$= \cosh 1 - 2\left[\sinh 1 - \cosh x \Big|_0^1 \right]$$

$$= \cosh 1 - 2 \sinh 1 + 2(\cosh 1 - 1)$$

$$= 3\cosh 1 - 2\sinh 1 - 2$$

$$= 3\left(\frac{e^x + e^{-x}}{2}\right) - (e^x - e^{-x}) - 2$$

$$= \left(\frac{3}{2} - 1\right)e^x + \left(\frac{3}{2} + 1\right)e^{-x} - 2$$

$$= \frac{1}{2}e^x + \frac{5}{2}e^{-x} - 2.$$