Domande / Commenti / Suggerimenti ?

- <u>Segnatura</u> di una forma bilineare simmetrica reale.

$b : V \times V \to \mathbb{R}$ simmetrica, bilineare.

$\mathcal{B} = \{v_1, \dots, v_n\}$: base ortogonale di (V, b).

$$\mathcal{B} = \mathcal{B}^+ \cup \mathcal{B}^- \cup \mathcal{B}^0,$$

$$\mathcal{B}^+ = \{ v_i \in \mathcal{B} \mid v_i^2 > 0 \}$$

$$\mathcal{B}^- = \{ v_i \in \mathcal{B} \mid v_i^2 < 0 \}$$

$$\mathcal{B}^0 = \{ v_i \in \mathcal{B} \mid v_i^2 = 0 \}$$

$$sg(b) = (|\mathcal{B}^+|, |\mathcal{B}^-|).$$

$\underline{Es}$: $V = \mathbb{R}^3$, $b = b_A$ $\quad A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$. $\det A = \det \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & 0 \end{pmatrix} = 2 \neq 0$

$\operatorname{Ker} b = \operatorname{Ker} A = \{0_V\} \Rightarrow b$ è non-degenere.

$e_1^2 = e_2^2 = e_3^2 = 0$, $\quad b_A(e_1, e_2) = a_{12} = 1 \neq 0 \Rightarrow u_1 = e_1 + e_2$ non è isotropo: $\quad u_1^2 = u_1^t A u_1 = (1\,1\,0) \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = (1\,1\,0) \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = 2 > 0.$

$\langle u_1 \rangle^\perp = \{X \in \mathbb{R}^3 \mid X^t A u_1 = 0\} = \left\{X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 \mid x_1 + x_2 + 2x_3 = 0\right\} = \left\langle \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} \right\rangle$

$u_2 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$: $\quad u_2^2 = (-1,1,0) \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} = (-1,1,0) \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = -2 < 0$

$\langle u_2 \rangle^\perp = \{X \in \mathbb{R}^3 \mid X^t A u_2 = 0\} = \left\{X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 \mid x_1 - x_2 = 0\right\}$

$\langle u_1 \rangle^\perp \cap \langle u_2 \rangle^\perp$: $\begin{cases} x_1 + x_2 + 2x_3 = 0 \\ x_1 - x_2 = 0 \end{cases}$: $\begin{pmatrix} 1 & 1 & 2 \\ 1 & -1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 \\ 0 & -2 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

$\langle u_1 \rangle^\perp \cap \langle u_2 \rangle^\perp = \left\langle \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \right\rangle \Rightarrow u_3 = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}.$

$\mathcal{B} = \{u_1, u_2, u_3\}$ è una base ortogonale di $(\mathbb{R}^3, b)$.

$u_3^2 = (-1,-1,1) \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} = (-1,-1,1) \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} = -2.$

$\Rightarrow \quad u_1^2 = 2 > 0, \quad u_2^2 = -2 < 0, \quad u_3^2 = -2 < 0$

$\operatorname{sg}(b) = (1, 2).$

Disuguaglianze di Cauchy-Schwarz (Richiami)

Sia (V, s) uno spazio euclideo. Siano $v, w \in V$. Allora

$$|s(v,w)| \leq \|v\| \|w\|.$$

In particolare: se $v \neq 0_V$ e $w \neq 0_V$

$$-1 \leq \frac{s(v,w)}{\|v\| \|w\|} \leq 1$$

Def:

$$\cos(\widehat{vw}) := \frac{s(v,w)}{\|v\| \|w\|}.$$

Disuguaglianza Triangolare

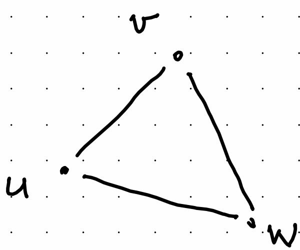
•) $\|v+w\| \leq \|v\| + \|w\| \qquad \forall v, w \in V.$

<u>dim</u>:

$$\|v+w\|^2 = s(v+w, v+w) = v^2 + w^2 + 2s(v,w)$$

$$\leq v^2 + w^2 + 2\,|s(v,w)| \underset{\substack{\uparrow \\ \text{C-S.}}}{\leq} v^2 + w^2 + 2\,\|v\|\,\|w\| =$$

$$= \|v\|^2 + \|w\|^2 + 2\,\|v\|\,\|w\| = (\|v\| + \|w\|)^2 \qquad \blacksquare.$$

•) 

$$\text{dist}(u, w) \leq \text{dist}(u, v) + \text{dist}(v, w)$$

<u>dim</u>:

$$\text{dist}(u,w) = \|u-w\| = \|u-v+v-w\|$$

$$\underset{\substack{\text{Dis.} \\ \text{Triang.}}}{\leq} \|u-v\| + \|v-w\|.$$

$$= \text{dist}(u,v) + \text{dist}(v,w).$$

Geometria analitica del piano

Consideriamo lo spazio euclideo $(\mathbb{R}^2, \cdot)$

$$X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad Y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \in \mathbb{R}^2$$

$$X \cdot Y := X^t Y = x_1 y_1 + x_2 y_2 = b_{\mathbb{1}_n}(X, Y).$$

La norma di X è

$$\|X\| = \sqrt{X \cdot X} = \sqrt{x_1^2 + x_2^2}$$

$$\|X\| \geq 0, \quad \|X\| = 0 \iff X = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = 0_{\mathbb{R}^2}, \quad \|\lambda X\| = \sqrt{\lambda^2}\,\|X\| = |\lambda|\,\|X\|$$

Es: $\left\| \begin{pmatrix} -3 \\ 6 \end{pmatrix} \right\| = \left\| -3 \begin{pmatrix} 1 \\ -2 \end{pmatrix} \right\| = 3 \left\| \begin{pmatrix} 1 \\ -2 \end{pmatrix} \right\| = 3\sqrt{5}.$

Un versore di $(\mathbb{R}^2, \cdot)$ è un vettore X t.c. $\|X\| = 1$

$$X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \text{ è un versore} \iff \sqrt{x_1^2 + x_2^2} = 1 \iff x_1^2 + x_2^2 = 1$$

Prop.: $X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ è un versore se e solo se $\exists\ \theta \in [0, 2\pi)$ t.c.

$x_1 = \cos\theta \quad , \quad x_2 = \sin\theta .$

Notazione: Dato $\theta \in [0, 2\pi)$ denotiamo

$$P_\theta = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} .$$

.) $P_{\theta + 2k\pi} = P_\theta \qquad \forall k \in \mathbb{Z}$

.) $-P_\theta = P_{\theta + \pi}$

.) $P_\theta \cdot P_\mu = 0 \iff \cos\theta\cos\mu + \sin\theta\sin\mu = 0$

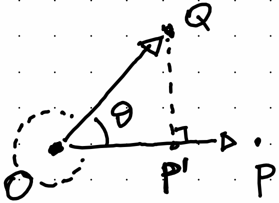
$\iff \cos(\theta)\cos(-\mu) - \sin\theta\sin(-\mu) = 0$

$\iff \cos(\theta - \mu) = 0$

$\iff \theta - \mu \in k\frac{\pi}{2}$ per qualche $k \in \mathbb{Z}$.

Rappresentazione grafica di $(\mathbb{R}^2, \cdot)$

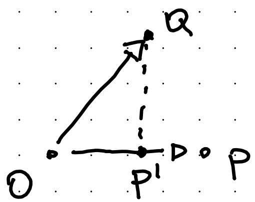
Sia $V = \mathcal{V}_0^2$. Il prodotto scalare standard di $\mathcal{V}_0^2$ è

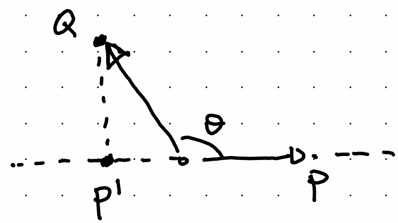


$$\vec{OP} \cdot \vec{OQ} := |\overline{OQ}|\,|\overline{OP}| \cos\theta$$

È ben definito, $\cos\theta = \cos(2\pi - \theta)$.

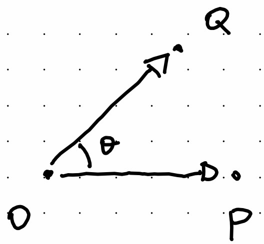
(1)



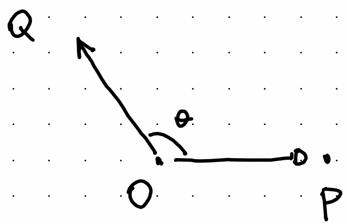


$|\overline{OP'}| = |\overline{OQ}| \cos\theta$ $|\overline{OP'}| = -|\overline{OQ}| \cos\theta$

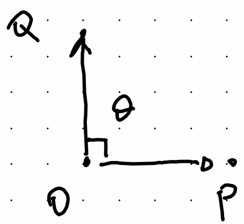
$$\vec{OP} \cdot \vec{OQ} = \begin{cases} |\overline{OP}|\,|\overline{OP'}| & \text{nel caso (1)} \\ -|\overline{OP}|\,|\overline{OP'}| & \text{nel caso (2).} \end{cases}$$



$$\vec{OP}\cdot\vec{OQ} > 0 \iff \theta \text{ acuto}$$

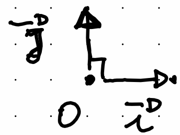


$$\vec{OP}\cdot\vec{OQ} < 0 \iff \theta \text{ ottuso.}$$



$$\vec{OP}\cdot\vec{OQ} = 0 \iff \theta = \frac{\pi}{2}.$$

Sia $\mathcal{B} = \{\vec{i}, \vec{j}\}$ la base ortonormale di $(\mathcal{V}_O^2, \cdot)$ data da



$$|\vec{i}| = |\vec{j}| = 1$$

$$F_{\mathcal{B}} : \mathcal{V}_O^2 \xrightarrow{\simeq} \mathbb{R}^2$$

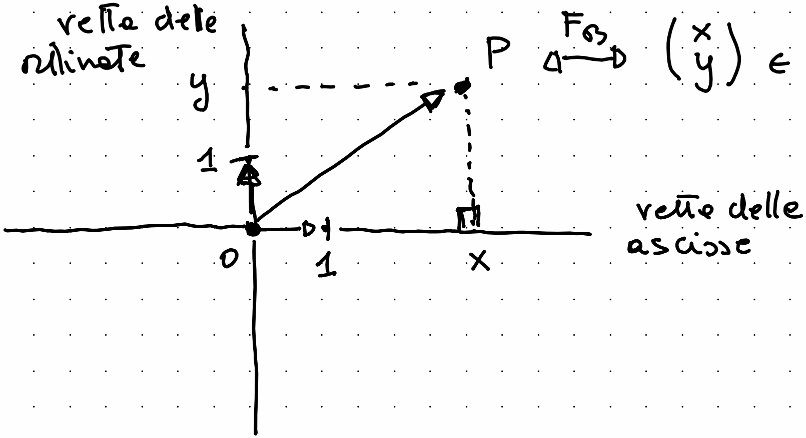
È un isomorfismo di spazi euclidei :

$$F_{\mathcal{B}}(\vec{OP}) \cdot F_{\mathcal{B}}(\vec{OQ}) = \vec{OP} \cdot \vec{OQ}$$

Infatti, se $\vec{OP} \neq \vec{OO}$ e $\vec{OQ} \neq \vec{OO}$ allora

$$F_{\mathcal{B}}(\vec{OP}) \cdot F_{\mathcal{B}}(\vec{OQ}) = \|F_{\mathcal{B}}(\vec{OP})\| \, \|F_{\mathcal{B}}(\vec{OQ})\| \cos\theta$$

$$= |\vec{OP}| \, |\vec{OQ}| \cos\theta = \vec{OP} \cdot \vec{OQ} .$$



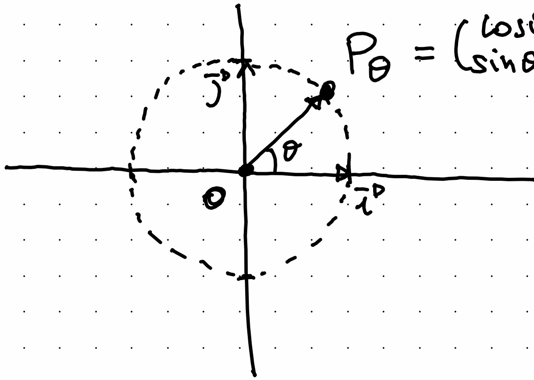
$P \xleftrightarrow{F_{\mathcal{B}}} \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2$

Da adesso in poi identifichiamo $(\mathbb{R}^2, \cdot)$ con $(\mathcal{V}_0^2, \cdot)$ rispetto alla base $\mathcal{B} = \{\vec{\imath}, \vec{\jmath}\}$.

$\Rightarrow$ Possiamo fare i disegni.

Circonferenza unitaria : I versori di $(\mathbb{R}^2, \cdot)$ giacciono sulla circonferenza

$$x^2+y^2=1$$



$$P_\theta = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

$$\cos\theta = P_\theta \cdot \vec{i} = P_\theta \cdot e_1$$

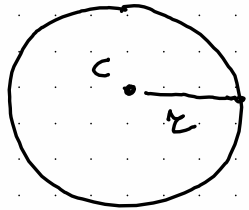
Circonferenze: Una circonferenza di centro $C=\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \in \mathbb{R}^2$ e raggio $r>0$ è il luogo dei punti che si trovano a distanza r da C:

$$\mathcal{C}(C,r) = \{ X \in \mathbb{R}^2 \mid \|X-C\| = r \}$$
$$= \{ X \in \mathbb{R}^2 \mid \|X-C\|^2 = r^2 \}$$
$$= \{ X = \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \mid (x-c_1)^2 + (y-c_2)^2 = r^2 \}$$

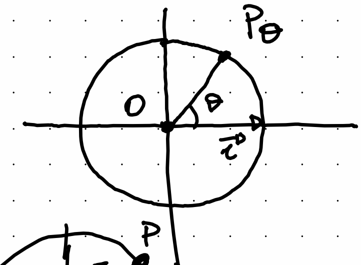
$\mathcal{C}(C,r)$: $(x-c_1)^2 + (y-c_2)^2 = r^2$: Eq. cartesiana di $\mathcal{C}(C,r)$.

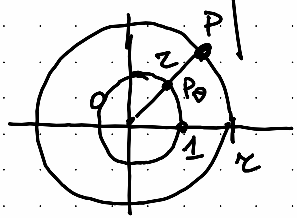
$\mathcal{C}(C,r)$: $x^2 + y^2 - 2c_1 x - 2c_2 y + (c_1^2 + c_2^2 - r^2) = 0$

Es: $\mathcal{C}\left(\begin{pmatrix} 1 \\ 2 \end{pmatrix}, 2\right)$: $x^2 + y^2 - 2x - 4y + 1 = 0$

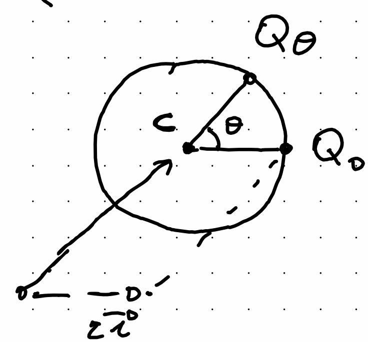
$\mathcal{C}(c,r) =$ 

$$\mathcal{C}(O,1) = \{\text{versori di } (\mathbb{R}^2,\cdot)\} = \{P_\theta \mid \theta\in[0,2\pi)\}$$

$=$ 

$\mathcal{C}(O,r) =$ 

$$= \left\{ rP_\theta = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix} \;\middle|\; \theta\in[0,2\pi) \right\}$$

$$\mathcal{C}(c,r) = \{\underbrace{c + rP_\theta}_{Q_\theta} \mid \theta\in[0,2\pi)\} =$$ 

Eq. parametriche di $\mathcal{C}(c,r)$.

Es: Trovare un' equazione parametrica della circonferenza

$$\mathcal{C}: \quad x^2 + y^2 - 3x + 4y + 4 = 0$$

Sol.: Completiamo i quadrati:

$$\left(x^2 - 2 \cdot \frac{3}{2} x + \frac{9}{4} - \frac{9}{4}\right) + \left(y^2 + 2 \cdot 2y + 4\right) = 0$$

$$\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} + (y+2)^2 = 0$$

$$\Rightarrow \quad \left(x - \frac{3}{2}\right)^2 + (y+2)^2 = \frac{9}{4}$$

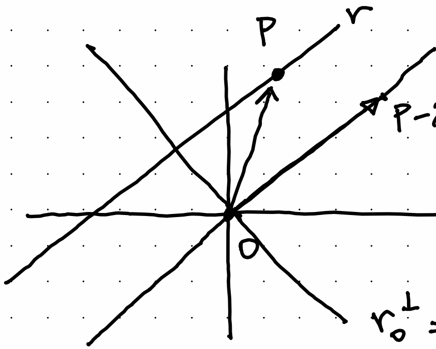
$$\Rightarrow \quad \mathcal{C} = \mathcal{C}\left(\begin{pmatrix} 3/2 \\ -2 \end{pmatrix}, \frac{3}{2} \right) \qquad C = \begin{pmatrix} 3/2 \\ -2 \end{pmatrix} \quad r = \frac{3}{2} = \sqrt{\frac{9}{4}}.$$

$$\mathcal{C} = \left\{ \begin{pmatrix} 3/2 \\ -2 \end{pmatrix} + \frac{3}{2} \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \;\middle|\; \theta \in [0, 2\pi) \right\}.$$

Vettori direttori e normali ad una retta

Sia $r: ax+by=c$ una retta di $\mathbb{R}^2$.

Un vettore direttore a r è $v = \begin{pmatrix} -b \\ a \end{pmatrix}$



$r_0 : ax+by=0 \qquad r_0 = \left\langle \begin{pmatrix} -b \\ a \end{pmatrix} \right\rangle$

I versori o coseni direttori di r sono $\pm \dfrac{1}{\sqrt{a^2+b^2}} \begin{pmatrix} -b \\ a \end{pmatrix}$

$r_0^\perp = \left\langle \begin{pmatrix} a \\ b \end{pmatrix} \right\rangle.$

La retta $\left\langle \begin{pmatrix} a \\ b \end{pmatrix} \right\rangle$ è ortogonale a r:

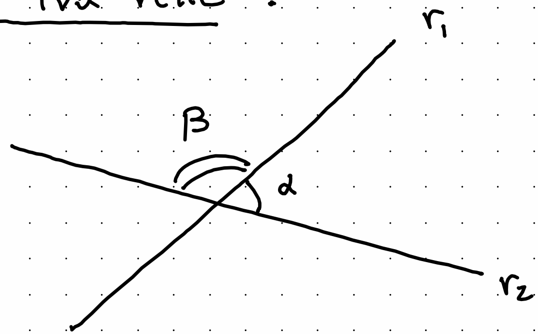
$P = \begin{pmatrix} x \\ y \end{pmatrix} \in r$ allora $P - X_0 = \begin{pmatrix} x' \\ y' \end{pmatrix} \in r_0$ dove $X_0 \in r$.

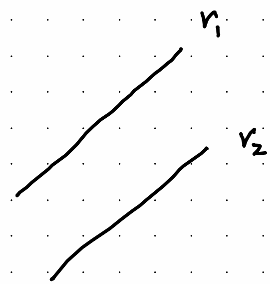
e quindi soddisfa $\begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \end{pmatrix} = 0$.

$\pm \dfrac{1}{\sqrt{a^2+b^2}} \begin{pmatrix} -b \\ a \end{pmatrix}$ sono i versori normali della retta.

Se $U = X_0 + U_0$ è un s. sp. affine, il sottospazio vettoriale $U_0^{\perp}$ è ortogonale a U_0 e quindi anche a U.

Angolo tra rette :





$\alpha + \beta = \pi$.

L'angolo tra r_1 e r_2 è quello acuto:

$r_1 = P + \langle v \rangle$ $\quad r_2 = Q + \langle w \rangle$, l'angolo $\widehat{r_1 r_2} \in [0, \frac{\pi}{2}]$ t.c.

$$\cos \widehat{r_1 r_2} = |\cos(v,w)| = \left| \frac{v \cdot w}{\|v\| \, \|w\|} \right|$$

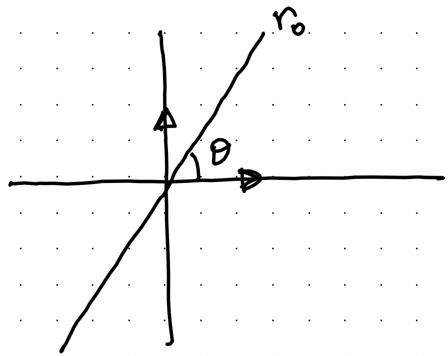
Pendenza o coefficiente angolare di una retta

$r: \; ax + by = c$ non parallela a $x = 0$ (asse delle ordinate) $\Leftrightarrow b \neq 0$

$$\Rightarrow \quad r: \; y = -\frac{a}{b}x + c$$

Il numero $-\frac{a}{b} = m$ si chiama la pendenza o coefficiente angolare di r.

$r_0: \; ax + by = 0 \qquad r_0 = \left\langle \begin{pmatrix} -b \\ a \end{pmatrix} \right\rangle = \left\langle \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \right\rangle.$



$$m = -\frac{a}{b} = \frac{\sin\theta}{\cos\theta} = Tg(\theta)$$

$$\boxed{r: \; y = Tg(\theta)\, x + c}$$

pendenza.

<u>Prop.</u> : Se $r_1 : y = m_1 x + q_1$ $\quad r_2 : y = m_2 x + q_2$.
Allora l'angolo α tra r_1 e r_2 è t.c.

$$\operatorname{tg}(\alpha) = \frac{|m_1 - m_2|}{|1 + m_1 m_2|}$$

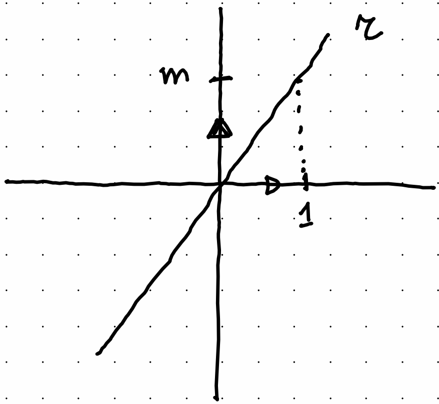
<u>Es</u>: Calcolare l'angolo tra

$$r_1 : 3x + y + 5 = 0 \qquad r_2 : 2x - y + 1 = 0$$

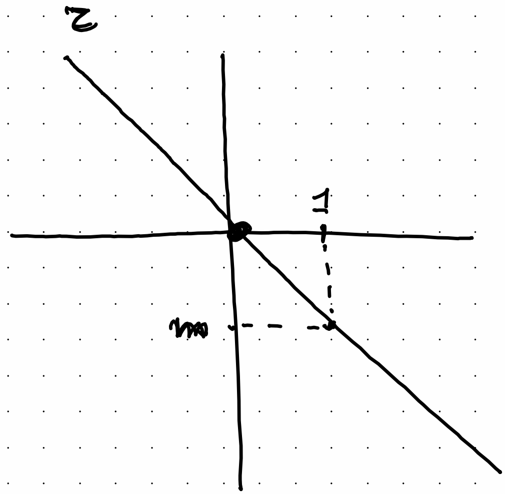
<u>Sol.</u>: $r_1 : y = -3x - 5$ $\qquad r_2 : y = 2x + 1$

$$\operatorname{tg}(\widehat{r_1 r_2}) = \frac{|-3-2|}{|1 + (-3)(2)|} = \frac{|-5|}{|-5|} = 1 \Rightarrow \widehat{r_1 r_2} = \frac{\pi}{4}$$

OSS: $r: y = mx$



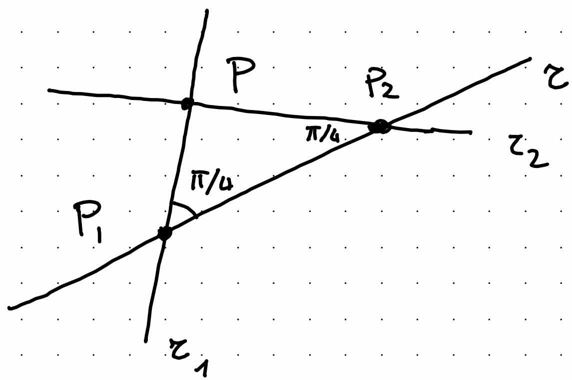
$m > 0$



$m < 0$

Es: Sia $r: 2x+y=1$ e $P=\begin{pmatrix}2\\2\end{pmatrix}$.
Trovare le rette passanti per P e che formano un angolo di $\frac{\pi}{4}$ con r, r_1 ed r_2.
$P_1 = r_1 \cap r$ $\quad P_2 := r_2 \cap r$. Calcolare area e perimetro del triangolo di vertici P, P_1 e P_2.

Sol.:



La pendenza di r è -2. $\quad r_1: y = m_1 x + q_1$, $\; r_2: y = m_2 x + q_2$

$$1 = \operatorname{Tg}(\widehat{r r_i}) = \frac{|-2-m|}{|1-2m|} \iff 1-2m = -2-m \text{ oppure } 1-2m = 2+m$$

$$\Rightarrow m = 3 \text{ oppure } m = -\frac{1}{3}$$

$r_1 : \; y = 3x + q_1 \qquad r_2 : \; y = -\frac{1}{3}x + q_2$

Imponiamo il passaggio per $P = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

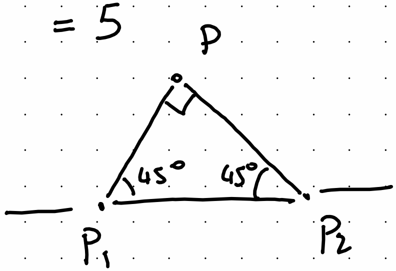
$r_1 : \; y = 3x - 4 \qquad r_2 : \; y = -\frac{1}{3}x + \frac{8}{3}$

$r_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix} + \left\langle \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right\rangle \qquad r_2 = \begin{pmatrix} 2 \\ 2 \end{pmatrix} + \left\langle \begin{pmatrix} 1 \\ -1/3 \end{pmatrix} \right\rangle$

$P_1 = r \cap r_1 = \dots = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad P_2 = r \cap r_2 = \dots = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

$$\text{Area}\left(\overset{\triangle}{PP_1P_2}\right) = \frac{1}{2} \left| \det \left(P_1 - P \,|\, P_2 - P \right) \right| = \frac{1}{2} \left| \det \begin{pmatrix} -1 & -3 \\ -3 & 1 \end{pmatrix} \right| = \frac{1}{2} |-10|$$

$= 5$



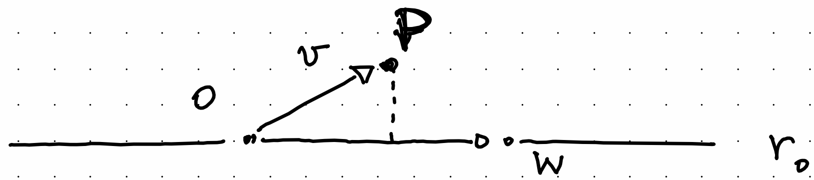
$$\text{Perimetro} = \|P_1 - P\| + \|P_2 - P\| + \|P_2 - P_1\| =$$

$$= \sqrt{10} + \sqrt{10} + \sqrt{4+16}$$

$$= 2\sqrt{10} + \sqrt{20}$$

Distanza punto-retta

$v, w \in \mathbb{R}^2, \quad w \neq 0_{\mathbb{R}^2}$



$$\operatorname{dist}(P, r_0) = \| P - \operatorname{pr}_w(v) \|$$

$$= \left\| P - \frac{v \cdot w}{w \cdot w} w \right\| \quad = \text{distanza punto-retta per } O.$$

$$r = Q + \langle w \rangle$$

invarianza per traslazioni

$$\operatorname{dist}(P, r) = \min_{t \in \mathbb{R}} \operatorname{dist}(P, Q + tw) = \min_{t \in \mathbb{R}} \operatorname{dist}(P - Q, tw)$$

$$= \| (P-Q) - \operatorname{pr}_w(P-Q) \|.$$

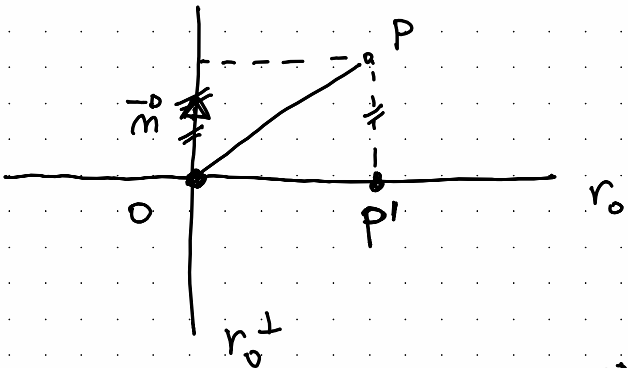
Es: $P = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ $r = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \langle \begin{pmatrix} 1 \\ 2 \end{pmatrix} \rangle$

$$\mathrm{dist}(P, r) = \left\| \begin{pmatrix} 2 \\ 1 \end{pmatrix} - \frac{\begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}}{\begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\|$$

$$= \left\| \begin{pmatrix} 2 \\ 1 \end{pmatrix} - \frac{4}{5} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\| =$$

$$= \left\| \begin{pmatrix} 6/5 \\ -3/5 \end{pmatrix} \right\| = \frac{3}{5} \left\| \begin{pmatrix} 2 \\ -1 \end{pmatrix} \right\| = \frac{3}{5} \sqrt{5} = \frac{3}{\sqrt{5}}$$

Se $r: ax+by=c$ allora $r^\perp = \left\langle \begin{pmatrix} a \\ b \end{pmatrix} \right\rangle$



$n = \begin{pmatrix} a \\ b \end{pmatrix}$

$P = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$

$$P - pr_r(P) = pr_n(P) = \frac{P \cdot \vec{n}}{\vec{n} \cdot \vec{n}} \vec{n} = \frac{ax_0 + by_0}{a^2+b^2} \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\operatorname{dist}(P, r) = \left\| \frac{(P-Q) \circ n}{n \circ n} n \right\| = \frac{|(P-Q) \cdot n|}{\|n\|^2} \|n\|$$

$$= \frac{|(P-Q) \cdot n|}{\|n\|} = \frac{|ax_0 + by_0 - c|}{\sqrt{a^2+b^2}}.$$

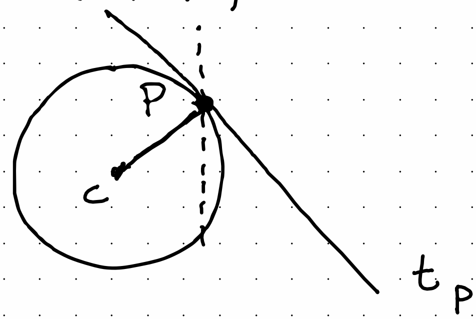
__Es__ : $P = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ $r: \ 2x - y = 1$

Allora

$$\mathrm{dist}(P, r) = \frac{|2 \cdot 3 - 1 \cdot 2 - 1|}{\sqrt{2^2 + (-1)^2}} = \frac{3}{\sqrt{5}}$$

Rette Tangenti ad una circonferenza

Sia $\mathcal{C} = \mathcal{C}(C, r)$, $P \in \mathcal{C}$. Sia t_P la retta tangente a $\mathcal{C}$ nel punto P.



t_P è la retta passante per P e ortogonale a $P-C$.

Se $P = C + r P_\theta \Rightarrow P - C = r P_\theta$

$t_P = P + \langle v \rangle$, $v \perp P_\theta = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \Rightarrow v = P_{\theta+\frac{\pi}{2}} = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$

$$t_P = P + \left\langle \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix} \right\rangle$$

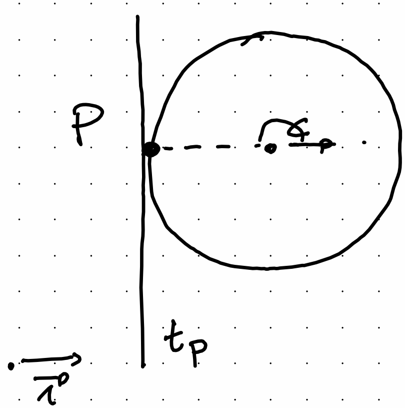
Es: $\mathcal{C}: x^2+y^2-3x+4y+4=0$, $P=\begin{pmatrix}0\\-2\end{pmatrix}$.

Trovare t_P.

Sol.:

$$\mathcal{C}: \left(x-\frac{3}{2}\right)^2+(y+2)^2=\frac{9}{4} \qquad C=\begin{pmatrix}3/2\\-2\end{pmatrix} \quad r=\frac{3}{2}$$

$$P=\begin{pmatrix}3/2\\-2\end{pmatrix}+\frac{3}{2}\begin{pmatrix}-1\\0\end{pmatrix}=C+rP_\pi$$



$$t_P=P+\left\langle\begin{pmatrix}0\\1\end{pmatrix}\right\rangle$$

$$t_P: \quad x=0$$