$u, v \in V \qquad \mathrm{Span}(u,v) = \{ t_1 u + t_2 v \mid t_1, t_2 \in \mathbb{K} \}$

$u \in \mathrm{Span}(u,v)$ Infatti,

$$u = u = 1u = 1u + \boxed{0_V} = \underline{1}\, u + \boxed{\underline{0} \cdot v}$$

$0 \in \mathbb{K}, \quad 0_V \in V$

Es: •) $V = \mathbb{R}^2$. $\quad 0 \in \mathbb{R}, \quad 0_V = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

•) $V = \mathbb{R}[x]_{\leq 1}, \quad 0 \in \mathbb{R}, \quad 0_V = 0 + 0x$

$\forall v \in V \quad 0 \cdot v = 0_V \quad : \qquad 0 \cdot v = (1-1)v = 1v - 1v$

prodotto per scalari

$$= v - v = 0_V$$

$$u = \begin{pmatrix} i \\ 0 \\ 1 \end{pmatrix} \qquad v = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \qquad w = \begin{pmatrix} 1+i \\ 1 \\ 0 \end{pmatrix}$$

Ci chiediamo: u è un multiplo di w?

Supponiamo che lo sia. Allora $\exists\, \alpha \in \mathbb{C}$ t.c.

$$u = \alpha w \iff \begin{pmatrix} i \\ 0 \\ 1 \end{pmatrix} = \alpha \begin{pmatrix} 1+i \\ 1 \\ 0 \end{pmatrix}$$

$$\iff \begin{pmatrix} i \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha + \alpha i \\ \alpha \\ 0 \end{pmatrix}$$

$$\iff \left.\begin{array}{l} i = \alpha + \alpha i \\ 0 = \alpha \\ 1 = 0 \end{array}\right] \quad \text{contraddizione!}$$

Quindi $u \notin Span(w)$. Quindi $Span(w) \boxed{\subsetneq} Span(u,v)$.

$A \subseteq B$: A è contenuto o uguale a B.

$A \subsetneq B$: A è contenuto ma non uguale a B.

$$\begin{array}{ccc} \{\alpha w \mid \alpha \in \mathbb{C}\} & & \{t_1 u + t_2 v \mid t_1, t_2 \in \mathbb{C}\} \\ \| & & \| \\ \mathrm{Span}(w) & \subset & \mathrm{Span}(u, v) \end{array}$$

[vuol dire che $\forall \alpha \in \mathbb{C}$ $\exists\, t_1, t_2 \in \mathbb{C}$ t.c.

$$\boxed{\alpha w = t_1 u + t_2 v}$$]

1) $u = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$, $v = \begin{pmatrix} 0 \\ i \\ -1 \end{pmatrix}$, $w = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$$\alpha w = \begin{pmatrix} \alpha \\ \alpha \\ \alpha \end{pmatrix} \stackrel{?}{=} t_1 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + t_2 \begin{pmatrix} 0 \\ i \\ -1 \end{pmatrix}$$

Se fosse così: $\alpha = 0 = t_1 = t_2$

$$\mathrm{Span}(w) \cap \mathrm{Span}(u, v) = \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$$

Quindi $\mathrm{Span}(w) \not\subset \mathrm{Span}(u, v)$.

$$u = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad v = \begin{pmatrix} 0 \\ i \\ -1 \end{pmatrix}, \quad w = \begin{pmatrix} \boxed{1} \\ 1 \\ 1 \end{pmatrix}$$

È vero che $\mathrm{Span}(w) \subseteq \mathrm{Span}(u,v)$?

<u>NO</u>: perché ad esempio $w \notin \mathrm{Span}(u,v)$

Se $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = t_1 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + t_2 \begin{pmatrix} 0 \\ i \\ -1 \end{pmatrix}$ allora nella prima ~~1=0~~ componente si avrebbe

$1 = 0$ contraddizione.

$(\mathbb{K}[x], +, \cdot)$ è uno spazio vettoriale.

"grado per grado"

1) Associatività: $p, q, r \in \mathbb{K}[x]$: $p(x) = \sum a_i x^i$, $q(x) = \sum b_i x^i$, $r(x) = \sum c_i x^i$

$$[(p+q)+r](x) \overset{\text{Def}}{=} \sum [(a_i+b_i)+c_i] x^i \overset{(\mathbb{K},+) \text{ è associativo.}}{=} \sum [a_i + (b_i+c_i)] x^i$$

$$\overset{\text{Def}}{=} [p + (q+r)](x)$$

2) El. neutro: $0_{\mathbb{K}[x]} = \sum_{i \geq 0} 0 x^i = 0 + 0x + 0x^2 + \ldots$

3)

4)

Ipotesi $\qquad\qquad\qquad\qquad$ Tesi

$$v+u = w+u \underset{\substack{\uparrow \\ \text{Assiomi} \\ \text{di} \\ \text{Sp. vettoriale}}}{\Longrightarrow} v = w$$

$+(-u) = +$ l'opposto di u

$$v \underset{\substack{\uparrow \\ \text{esistenza} \\ \text{di } 0_V}}{=} v+0_V \underset{\substack{\uparrow \\ \text{esistenza} \\ \text{dell'opposto} \\ \text{di } u}}{=} v+(u-u) \underset{\substack{\uparrow \\ \text{associatività}}}{=} (v+u)-u \underset{\substack{\uparrow \\ \text{Ipotesi}}}{=} (w+u)-u =$$

$$\underset{\substack{\uparrow \\ \text{assoc.}}}{=} w+(u-u) = w+0_V = w$$

$$\mathbb{K}^3 = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \;\middle|\; x_1, x_2, x_3 \in \mathbb{K} \right\}$$

$$= \left\{ \overbrace{x_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}} \;\middle|\; x_1, x_2, x_3 \in \mathbb{K} \right\}$$

$$= \mathrm{Span}\left(\underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}_{= e_1}, \underbrace{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}_{= e_2}, \underbrace{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}_{= e_3} \right)$$

generatori standard di $\mathbb{K}^n$.

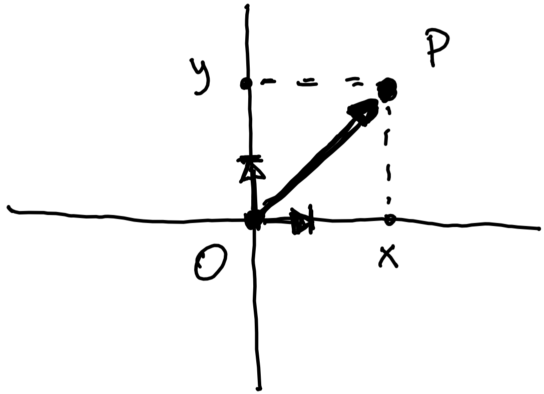
$$\mathbb{K}^n = \mathrm{Span}\,(e_1, e_2, \dots, e_n) \quad \text{dove} \quad e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \; e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

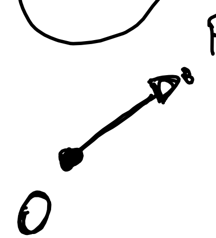
$$e_i := \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \leftarrow i$$

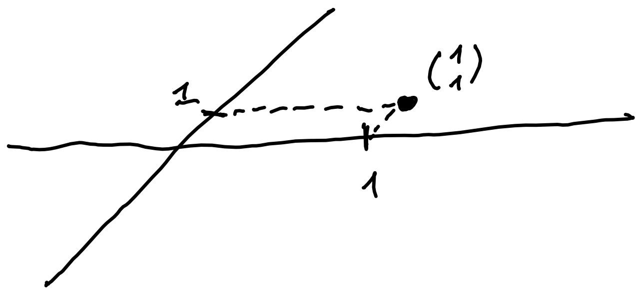
$\begin{pmatrix} x \\ y \end{pmatrix}$

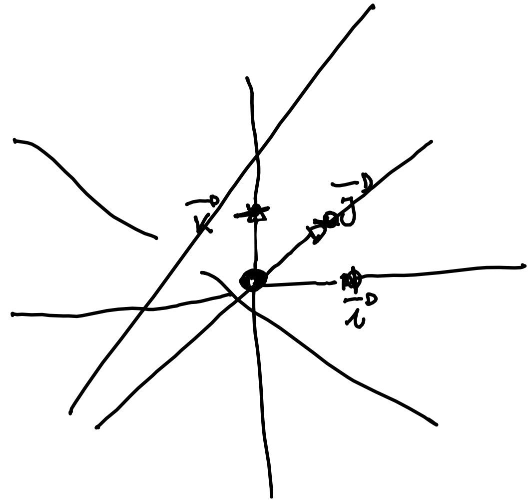
Vettori geometrici del piano

V_O^2









$\mathbb{R}^3$

$$2A + X = 2B$$

$$\Rightarrow X = 2B - 2A$$

$$\boxed{A + X = B} \Rightarrow X = B - A$$

$\Downarrow$ Esistenza di $-A$

$$(A + X) + (-A) = B + (-A)$$

$\Downarrow$ Assoc.

$$A + (X - A) = B - A \overset{\text{comm}}{\Rightarrow} A + (-A + X) = B - A$$

$$\underset{\text{assoc.}}{\Rightarrow} (A + (-A)) + X = B - A$$

$$\underset{\substack{\text{Esistenza} \\ \text{di el. inv.}}}{\Rightarrow} 0 + X = B - A \Rightarrow X = B - A$$