Eq. cartesiane e parametriche

$U: \ Ax = b \quad \subset \mathbb{K}^n \qquad \mathrm{rg}\,A = r$

$\Rightarrow$ La forma parametrica di U è

$U = X_0 + \mathrm{Ker}\,A$ dove X_0 è una soluzione del sistema.

Es: $U: \begin{cases} 2x + y = 1 \\ x - y = 2 \end{cases}$ $\left(\begin{array}{cc|c} 2 & 1 & 1 \\ 1 & -1 & 2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -1 & 2 \\ 2 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -1 & 2 \\ 0 & 3 & -3 \end{array}\right)$

$\sim \left(\begin{array}{cc|c} 1 & -1 & 2 \\ 0 & 1 & -1 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -1 \end{array}\right)$

$U = \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}.$

Retta per 2 punti $P_0 \neq P_1 \subset \mathbb{R}^2$

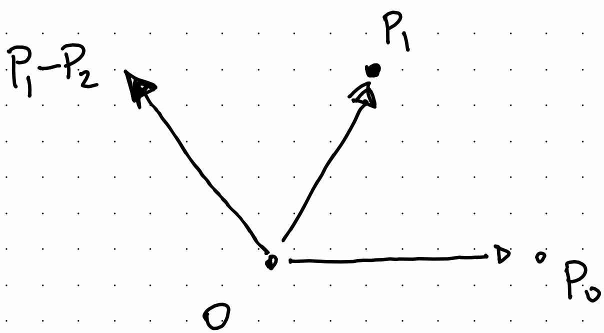
$$r = P_0 + \langle P_1 - P_0 \rangle .$$

$$r: \quad ax + by = c \qquad \text{dove} \qquad P_1 - P_0 = \begin{pmatrix} -b \\ a \end{pmatrix}$$

$$c = (a, b) \cdot P_0$$

$$\overrightarrow{OP_1} \qquad \overrightarrow{OP_2}$$

$$V_0^2 \underset{F_{\mathcal{B}}}{\simeq} \mathbb{R}^2$$

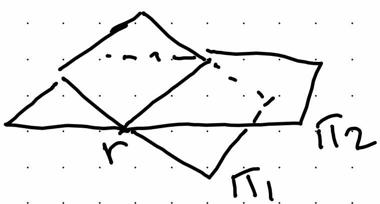


Posizione piano/piano in $\mathbb{R}^3$

$\pi_1: \; ax+by+cz=d \qquad \pi_2 = X_0 + \langle v_1, v_2 \rangle$

$A=(a,b,c) \qquad (Av_1 \mid Av_2) \qquad d-AX_0$

	$rg\,(Av_1 \mid Av_1)$	$rg\,(Av_1 \mid Av_2 \mid d-AX_0)$
$\pi_1 \equiv \pi_2$	0	0
$\pi_1 \,//\, \pi_2,\ \pi_1 \cap \pi_2 = \emptyset$	0	1
$\pi_1 \cap \pi_2 =$ retta	1	1



Determinante

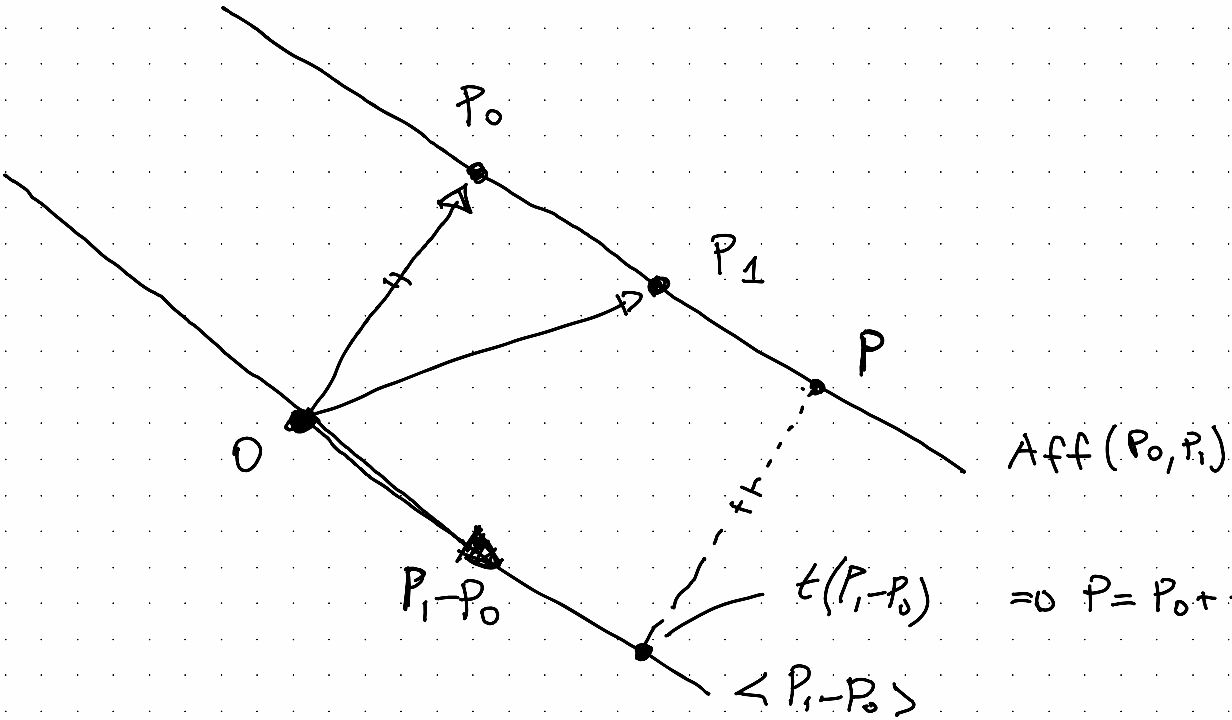
Sia $A \in Mat_{n\times n}(\mathbb{K})$. $\quad \det A \neq 0 \iff rg\, A = n$.

Infatti, $\quad \det A \neq 0 \iff rref(A) = \mathbb{1}_n \iff rg\, A = n$

Inviluppo affine

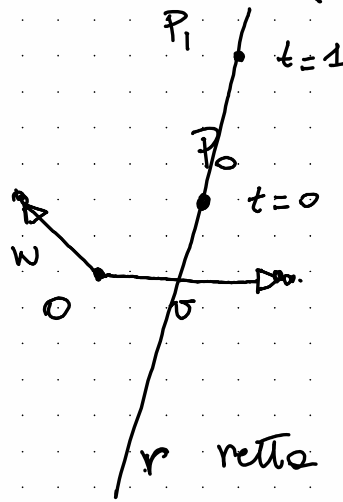
$$\mathrm{Aff}(P_0, \dots, P_n) = \{ x_0 P_0 + \dots + x_n P_n \mid x_0 + \dots + x_n = 1 \}.$$

$$= P_0 + \langle P_1 - P_0, \dots, P_n - P_0 \rangle$$



$\Rightarrow P = P_0 + t(P_1 - P_0)$

$\underline{Es}$: $\quad P_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad P_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$



base

$\mathcal{B} = \{v, w\} \subset \mathcal{V}_0^2$

$F_{\mathcal{B}} : \mathcal{V}_0^2 \longrightarrow \mathbb{R}^2$

$\begin{pmatrix} 1 \\ 1 \end{pmatrix} + t \begin{pmatrix} 2 \\ 3 \end{pmatrix} - t \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

r retta per P_0 e P_1

$$r = P_0 + \langle P_1 - P_0 \rangle = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \left\langle \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\rangle = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \end{pmatrix} \,\middle|\, t \in \mathbb{R} \right\}$$

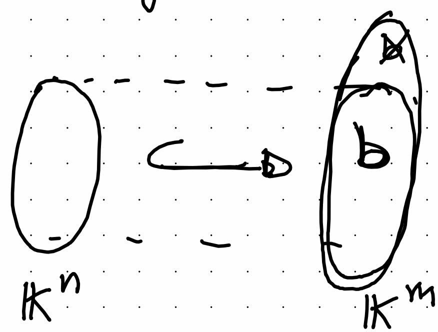
$$= \{ (1-t) P_0 + t P_1 \mid t \in \mathbb{R} \} = \mathrm{Aff}(P_0, P_1)$$

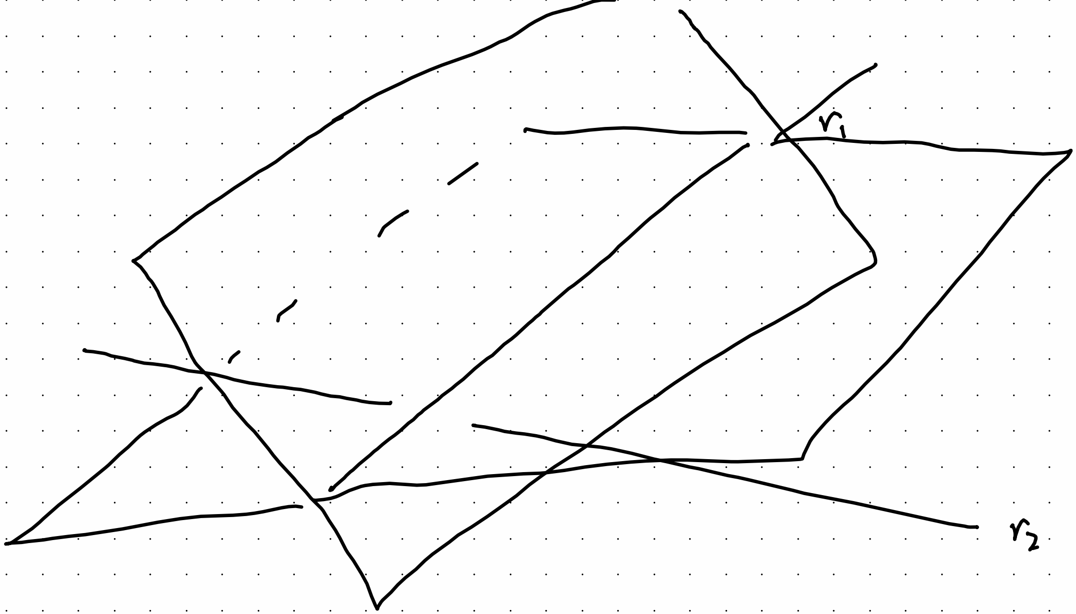
Sistemi non-singolari

Un sistema $AX=b$ si dice non-singolare se ammette un'unica soluzione. ($\Leftrightarrow \operatorname{Ker} A = \{0\}$ e $b \in \operatorname{Im} A$).

$AX=b$ è non-singolare $\forall b \Leftrightarrow A$ è invertibile.

S_A :





<u>Es</u>: $\pi: x+y+z=3$ $\quad r = \begin{pmatrix}1\\2\\3\end{pmatrix} + \left\langle \begin{pmatrix}1\\-1\\0\end{pmatrix} \right\rangle$

la giacitura di π è

$\pi_0: x+y+z=0$ $\quad r_0 = \left\langle \begin{pmatrix}1\\-1\\0\end{pmatrix} \right\rangle$

$\Rightarrow$ π ed r sono paralleli.

Es:

$$r_1 : \begin{cases} 2x+y-z=1 \\ x-y+2z=2 \end{cases} \qquad r_2 = \begin{pmatrix}1\\1\\1\end{pmatrix} + \left\langle \begin{pmatrix}1\\-1\\0\end{pmatrix} \right\rangle$$

Fascio di piani per r_1:

$$\pi_{a,b} : a\,(2x+y-z-1) + b\,(x-y+2z-2) = 0$$

$$x(2a+b) + y(a-b) + z(-a+2b) = a+2b$$

$\pi_{a,b}$ è parallelo a r_2 $\iff$ $\begin{pmatrix}1\\-1\\0\end{pmatrix} \in (\pi_{a,b})_0$

$\iff \quad 2a+b-(a-b)=0$

$\iff \quad a+2b=0$

$\iff \quad a=-2b \quad \iff \quad \pi_{a,b} = \pi_{-2,1}$

Il piano cercato è

$$x(-4+1)+y(-2-1)+z(2+2) = -2+2 \iff \pi_{a,b}:$$

$$\pi_{a,b} : -3x-3y+4z=0$$

Sottospazio di giacitura

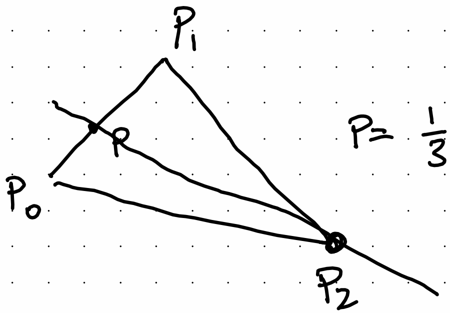
$$W = v + W_0 = v' + W_0'$$

$$\Rightarrow \quad W_0 = v' - v + W_0' \qquad \boxed{v' - v \in W_0'}. \text{ Infatti,}$$

$$v = v + 0_{W_0} \in v + W_0 = W = v' + W_0'$$

$$\Rightarrow \exists\, w_0' \in W_0' \text{ t.c. } \quad v = v' + w_0' \quad \Rightarrow \quad v - v' = w_0' \in W_0'$$

$$\Rightarrow \quad v' - v = -w_0' \in W_0'.$$



$$P = \frac{1}{3} P_0 + \frac{2}{3} P_1$$

$$P_2 + t \langle P - P_2 = \frac{1}{3} P_0 + \frac{2}{3} P_1 - P_2 \rangle$$

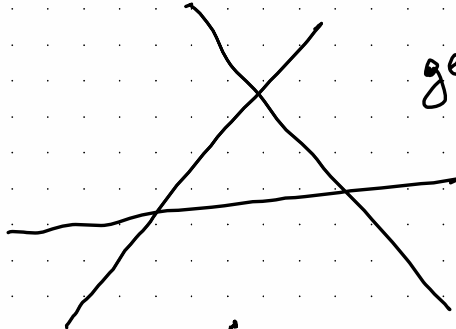
Esercizio:

$$W = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 \;\middle|\; x_1 - x_2 - x_3 = 1 \right\}$$

$$W: \quad x_1 - x_2 - x_3 = 1$$

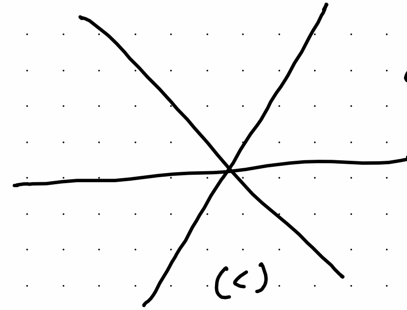
$$\Rightarrow \quad W = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \mathrm{Ker}\,(1, -1, -1)$$

$$= \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \left\langle \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\rangle.$$



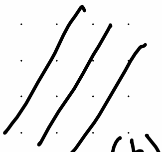
generiche

(d)

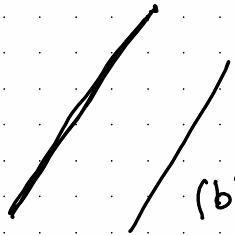


(c)

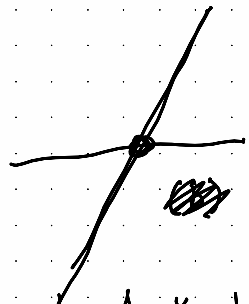
concorrenti



(b)



(b)



(c)

(a)

$r_1 : A_1 x = b_1 \qquad r_2 : A_2 x = b_2 \qquad r_3 : A_3 x = b_3$

$$\begin{cases} A_1 x = b_1 \\ A_2 x = b_2 \\ A_3 x = b_3 \end{cases}$$

	$\mathrm{rg}\begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix}$	$\mathrm{rg}\left(\begin{array}{c\|c} A_1 & b_1 \\ A_2 & b_2 \\ A_3 & b_3 \end{array}\right)$	
$r_1 \equiv r_2 \equiv r_3$	1	1	(a)
$r_1 \equiv r_2$, $r_3 // r_1$	1	2	(b)
concorrenti	2	2	(c)
generiche	2	3	(d)

Eq. cart. $\rightsquigarrow$ Eq. par.

$U: \; AX = b \quad \rightsquigarrow \quad U = X_0 + \operatorname{Ker} A$

Risolvere il sistema:

- Trovare una soluzione particolare X_0
- Trovare una base di $\operatorname{Ker} A$.

$(A|b) \rightsquigarrow \operatorname{rref}(A|b) = (R|c)$. $\qquad R = \operatorname{rref}(A)$.

$\operatorname{Ker} A = \operatorname{Ker} R = \langle \text{soluzioni-base} \rangle$

$RX = c \quad \rightsquigarrow \quad X_0.$

porre uguali a zero le variabili libere

col. dom. $\quad$ var. libere

Es: $U: \; x_1 + x_2 + x_3 = 1 \qquad A = (1\;1\;1) \qquad (A|b) = (\boxed{1}\;1\;1\,|\,1)$ è a scala ridotta

$$X_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \operatorname{Ker} A = \left\langle \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\rangle.$$

Eq. par $\rightsquigarrow$ Eq. cartesiane

$U = X_0 + \langle v_1, \dots, v_r \rangle \subset \mathbb{K}^n$ Per trovare eq. cartesiane di U:

1) $A = (v_1 | \dots | v_r) \rightsquigarrow A^t = \begin{pmatrix} v_1^t \\ \vdots \\ v_r^t \end{pmatrix} \rightsquigarrow \operatorname{Ker} A^t = \langle Y_1^t, \dots, Y_{n-r}^t \rangle$

$$\Rightarrow \quad U: \begin{cases} Y_1 X = Y_1 X_0 \\ Y_2 X = Y_2 X_0 \\ \vdots \\ Y_{n-r} X = Y_{n-r} X_0 \end{cases}$$

Es: $U = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \left\langle \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\rangle$: $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \rightsquigarrow A^t = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

$$\operatorname{Ker} A^t = \operatorname{Ker} \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} = \operatorname{Ker} \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 1 \end{pmatrix} = \operatorname{Ker} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix} = \left\langle \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \right\rangle$$

$\Rightarrow \quad U: \quad -X_1 + X_2 + X_3 = (-1, 1, 1) \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 1$ Eq. cart. di U.

2) $(A | X) \rightsquigarrow \left(\begin{array}{c|c} * & * \\ \hline 0 & B \end{array} \right) \Rightarrow U: \ BX = BX_0$

Es: $\left(\begin{array}{cc|c} 1 & 1 & X_1 \\ 1 & 0 & X_2 \\ 0 & 1 & X_3 \end{array} \right) \rightsquigarrow \left(\begin{array}{cc|c} 1 & 1 & X_1 \\ 0 & -1 & X_2 - X_1 \\ 0 & 1 & X_3 \end{array} \right) \sim \left(\begin{array}{cc|c} 1 & 1 & X_1 \\ 0 & -1 & * \\ \hline \boxed{0 \quad 0} & & \boxed{X_3 + X_2 - X_1} \end{array} \right) \rightsquigarrow X_3 + X_2 - X_1 = 1$