Esercizi Settimanali

$(a+b)(c+d) = ac + bd + \dots$
$\neq ac + bd$

Es 1:

$-\begin{pmatrix}0\\1\\0\end{pmatrix}$

a) $\cancel{\{X \in \mathbb{R}^3 \mid x_1+x_2+x_3=1\}}$

b) $\{X \in \mathbb{R}^3 \mid x_1+x_2-x_3=0\}$ V

c) $\cancel{\{X \in \mathbb{R}^3 \mid x_1^2+x_2-x_3=0\}}$

d) $\cancel{\{X \in \mathbb{R}^3 \mid x_1+x_2-x_3=0,\ x_1\geq 0\}}$ X $\ni \begin{pmatrix}1\\1\\2\end{pmatrix}$ ma $-\begin{pmatrix}1\\1\\2\end{pmatrix} \notin$.

e) $\cancel{\{X \in \mathbb{R}^3 \mid x_1\geq 0,\ x_2+x_3=0\}}$ $\ni \begin{pmatrix}1\\0\\0\end{pmatrix}$, ma $-\begin{pmatrix}1\\0\\0\end{pmatrix} \notin$.

f) $\cancel{\{X \in \mathbb{R}^3 \mid x_1x_2x_3=0\}}$ $\ni \begin{pmatrix}0\\1\\1\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix} \sim \begin{pmatrix}1\\2\\1\end{pmatrix} \notin$

g) $\{X \in \mathbb{R}^3 \mid x_1+x_2-x_3=0,\ x_2+x_3=0\}$ V

h) $\cancel{\{X \in \mathbb{R}^3 \mid x_1+x_2-x_3=0,\ x_2+x_3=1\}}$

$$\to \left\{X \,\middle|\, \begin{matrix} x_1 = -x_2+x_3 \\ x_2 = -x_3 \end{matrix}\right\} = \left\{X \,\middle|\, \begin{matrix} x_2 = -x_3 \\ x_1 = 2x_3 \end{matrix}\right\} = \left\{\begin{pmatrix}2x_3\\-x_3\\x_3\end{pmatrix} \,\middle|\, x_3 \in \mathbb{R}\right\}$$

$$= \left\{x_3\begin{pmatrix}2\\-1\\1\end{pmatrix} \,\middle|\, x_3 \in \mathbb{R}\right\} = \left\langle \begin{pmatrix}2\\-1\\1\end{pmatrix}\right\rangle. \quad V$$

ES 2: Calcolare $U \cap W$:

1) $U = \langle u, u+v \rangle$, $W = \langle 2u+v \rangle$

2) $U = \langle u+v, u-v \rangle$, $W = \langle u, v \rangle$

3) $U = \langle u+v \rangle$, $W = \langle u+2v \rangle$

$U = W \Leftrightarrow$ u e v sono lin. dip. $U \cap W = \{0_V\}$.

3) $s(u+v) = t(u+2v)$.

$u+v = t(u+2v)$

$u+v = tu + 2tv$

$(1-t)u = (2t-1)v$

$u = \frac{(2t-1)}{1-t} v$

Sol.: 1) $u + (u+v) = 2u + W \in W \cap \cancel{W} \Rightarrow W \subset U$.

$U = W \Leftarrow U \subset W \Leftarrow u = t(2u+v)$, $u+v = s(2u+v)$

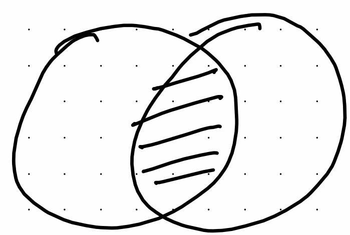
$u = 2tu + tv \Rightarrow \boxed{(2t-1)u = tv}$ (1)

$v = (2s-1)u + sv \Rightarrow (s-1)v = (1-2s)u$ (2)

Se $t \neq 0$ allora da (1) $v = \frac{2t-1}{t} u$ e quindi $v \in \langle u \rangle$

Se $t = 0$ allora $-u = 0_V \Rightarrow u = 0_V$

$\Leftarrow$ u e v sono lin. dip.

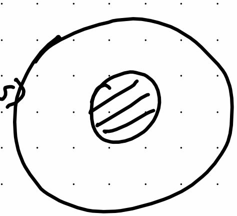


2) $U \subset W \Rightarrow U \cap W = U$

$\langle u, v \rangle = \langle u+v, v \rangle = \langle u+v, u-v \rangle$

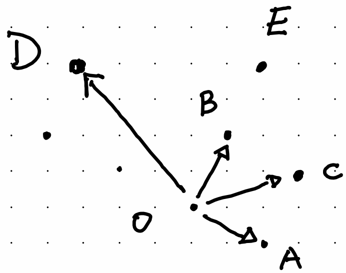
$u - v = u + v - 2v$

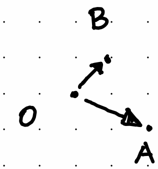
$\Rightarrow U = W$.



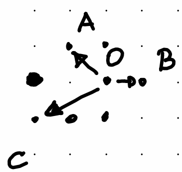
Es3 : 1) Calcolare $\overrightarrow{OA}+\overrightarrow{OB} = \overrightarrow{OC}$, $(-2)\overrightarrow{OA}+\overrightarrow{OB} = \overrightarrow{OD}$, $2\overrightarrow{OB} = \overrightarrow{OE}$

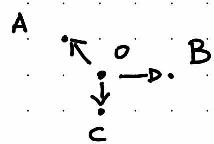
1)





2) $(\overrightarrow{OA}+\overrightarrow{OB})+\overrightarrow{OC} = \overrightarrow{OA}+(\overrightarrow{OB}+\overrightarrow{OC})$, $2(\overrightarrow{OA}+\overrightarrow{OB}) = 2\overrightarrow{OA}+2\overrightarrow{OB}$





$\underline{\text{Es 4}}$: $\{v_1, v_2, v_3\}$ è lin. Ind. ?

1) $v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $v_3 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ DIP. (Teo fond. Ind. Lin.)

2) $v_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $v_3 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ IND.

3) $v_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}$, $v_3 = \begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$ IND.

4) $v_1 = 1 + x$, $v_2 = 1 + x - x^2$, $v_3 = 1 + x + x^3$ IND.

5) $v_1 = \sin(x)$, $v_2 = \sin(2x)$, $v_3 = \sin(3x)$ IND.

$t_1 \cos x + 2 t_2 \cos(2x) + 3 t_3 (\cos 3x) = 0$

5): $t_1 \sin x + t_2 \sin(2x) + t_3 \sin(3x) = 0 \quad \forall x \in \mathbb{R}$

$t_1 = 0$

$x = \frac{\pi}{2}$: $t_1 + 0 t_2 - t_3 = 0$

$x = \pi/3$: $t_1 \frac{\sqrt{3}}{2} + t_2 \frac{\sqrt{3}}{2} = 0$

$x = \pi/6$: $\frac{1}{2} t_1 + t_2 \frac{\sqrt{3}}{2} + t_3 = 0$

$$\Rightarrow \begin{cases} t_1 - t_3 = 0 \\ t_1 + t_2 = 0 \\ t_1 + \sqrt{3} t_2 + 2 t_3 = 0 \end{cases}$$

$t_1 = t_3$

$t_2 = -t_1 = -t_3$

$t_3 - \sqrt{3} t_3 + 2 t_3 = 0$

$(3 - \sqrt{3}) t_3 = 0 \Rightarrow t_3 = 0$

$t_1 = 0$

$\Uparrow$

$t_1 = 0$

Es 5 :

$$\mathcal{B}_1 = \left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}, \quad \mathcal{B}_2 = \left\{ \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right\}, \quad \mathcal{B}_3 = \left\{ \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \end{pmatrix} \right\}$$

$$v_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 7 \\ -7 \end{pmatrix}, \quad v_4 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Sol. :

$$v_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix} = t_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + t_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} \iff \begin{cases} 2t_1 + t_2 = 1 \\ t_1 + t_2 = 3 \end{cases} \iff \begin{cases} t_1 + 3 = 1 \\ t_1 + t_2 = 3 \end{cases}$$

$$\iff \begin{array}{l} t_1 = -2 \\ t_2 = 3 - t_1 = 5 \end{array} \qquad v_1 = -2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + 5 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$ verifichiamo

$$F_{\mathcal{B}_1}(v_1) = \begin{pmatrix} -2 \\ 5 \end{pmatrix}; \quad F_{\mathcal{B}_2}(v_1) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \quad F_{\mathcal{B}_3}(v_1) = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$F_{\mathcal{B}_1}(v_2) = \begin{pmatrix} 3 \\ -4 \end{pmatrix}; \quad F_{\mathcal{B}_2}(v_2) = \begin{pmatrix} -8/3 \\ 7/3 \end{pmatrix}; \quad F_{\mathcal{B}_3}(v_2) = \begin{pmatrix} -3/2 \\ -1/2 \end{pmatrix}$$

$$F_{\mathcal{B}_1}(v_3) = \begin{pmatrix} 14 \\ -21 \end{pmatrix}; \quad F_{\mathcal{B}_2}(v_3) = \begin{pmatrix} -35/3 \\ 28/3 \end{pmatrix}; \quad F_{\mathcal{B}_3}(v_3) = \begin{pmatrix} -7 \\ 0 \end{pmatrix}$$

$$F_{\mathcal{B}_1}(v_4) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}; \quad F_{\mathcal{B}_2}(v_4) = \begin{pmatrix} -1/3 \\ 2/3 \end{pmatrix}; \quad F_{\mathcal{B}_3}(v_4) = \begin{pmatrix} 0 \\ -1 \end{pmatrix}.$$