Correzione Es. Settimanali:

Es 1: $V = \mathbb{R}[x]_{\leq 3}$,

$\mathcal{B}_1 = \{1-x,\ 1+x,\ x+x^2,\ 2x-x^2+2x^3\}$, $\mathcal{B}_2 = \{1,\ x-1,\ (x-1)^2,\ (x-1)^3\}$.

$p(x) = 2-3x+2x^2-x^3 \in V$.

$$p(x) = a_0(1-x) + a_1(1+x) + a_2(x+x^2) + a_3(2x-x^2+2x^3)$$

$$p(0) = a_0 + a_1 \qquad\qquad a_0 + a_1 = 2$$

$$p(x) = a_0(1-x) + a_1(1+x) + a_2 x(1+x) + a_3 x(2-x+2x^2)$$

$$= a_0(1-x) + (a_1 + a_2 x)(1+x) + a_3 x(2-x+2x^2)$$

$$a_0 = \frac{11}{4},\quad a_1 = -\frac{3}{4},\quad a_2 = \frac{3}{2},\quad a_3 = -\frac{1}{2} \qquad \checkmark$$

$$\mathcal{B}_2 = \{1, x-1, (x-1)^2, (x-1)^3\}.$$

$$p(x) = 2-3x+2x^2-x^3 = a_0 + a_1(x-1) + a_2(x-1)^2 + a_3(x-1)^3$$

$$a_0 \overset{!}{=} p(1) = 2-3+2-1 = 0$$

$$p(x) - a_0 = (x-1)\underbrace{\left[a_1 + a_2(x-1) + a_3(x-1)^2\right]}_{q_1(x)}$$

$$a_1 = -2$$

$$p(x) - a_0 - a_1(x-1) = (x-1)^2 \underbrace{\left[a_2 + a_3(x-1)\right]}_{q_2(x)}$$

$$p(x) - a_0 - a_1(x-1) = -x^3 + 2x^2 - 3x + 2 + 2(x-1)$$

$$= -x^3 + 2x^2 - 3x + 2x = -x^3 + 2x^2 - 3x + 2x \;\Big|\; x^2 - 2x + 1$$

$$\begin{array}{l|l} -x^3 + 2x^2 - 3x + 2x & x^2 - 2x + 1 \\ \hline -(-x^3 + 2x^2 - x) & -x \end{array}$$

Es 2:

$$U = \left\{ x \in \mathbb{R}^4 \;\middle|\; \begin{cases} x_1 - 2x_2 + x_4 = 0 \\ x_3 + x_4 = 0 \end{cases} \right\} \qquad W = \left\langle \begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\rangle$$

$$U = \left\{ x \;\middle|\; \begin{matrix} x_1 = 2x_2 - x_4 \\ x_3 = -x_4 \end{matrix} \right\} =$$

$$= \left\{ \begin{pmatrix} 2x_2 - x_4 \\ x_2 \\ -x_4 \\ x_4 \end{pmatrix} = x_2 \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix} \;\middle|\; x_2, x_4 \in \mathbb{R} \right\}$$

$$= \left\langle \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix} \right\rangle \qquad \dim U = 2 = \dim W$$

$$\Rightarrow \dim U \cap W \in \{0, 1, \cancel{2}\}$$

$$\dim U + W \in \{\cancel{2}, 3, 4\}$$

$$\dim U + W = \dim U + \dim W - \dim U \cap W$$

$$= 4 - \dim U \cap W$$

Es 2:

$$U = \left\{ x \in \mathbb{R}^4 \;\middle|\; \begin{cases} x_1 - 2x_2 + x_4 = 0 \\ x_3 + x_4 = 0 \end{cases} \right\} \qquad W = \left\langle \underbrace{\begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix}}_{\in U}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\rangle$$

$\Rightarrow \dim U \cap W = 1$

Una sua base è

$$\mathcal{B}_{U \cap W} = \left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix} \right\}$$

$$\mathcal{B}_U = \left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\} \qquad \mathcal{B}_W = \left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$\mathcal{B}_U \cup \mathcal{B}_W = \left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\} = \mathcal{B}_{U+W}$$

Es3:

$$U_1 = \left\{ X \in \mathbb{C}^3 \;\middle|\; \begin{matrix} x_1 + i x_2 = 0 \\ x_2 + i x_3 = 0 \end{matrix} \right\} \qquad U_2 = \left\{ x \in \mathbb{C}^3 \mid x_1 = -x_3 \right\}$$

$$U_1 = \left\{ X \;\middle|\; \begin{matrix} x_1 = -i x_2 \\ x_2 = -i x_3 \end{matrix} \right\} \qquad U_2 = \left\langle \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\rangle$$

$$= \left\{ X \;\middle|\; \begin{matrix} x_1 = -x_3 \\ x_2 = -i x_3 \end{matrix} \right\}$$

$$F_{\mathcal{B}_{U_1}} : U_1 \longrightarrow \mathbb{C}, \quad x\begin{pmatrix} -1 \\ -i \\ 1 \end{pmatrix} \longmapsto x$$

$$F_{\mathcal{B}_{U_3}} : \mathbb{C}^3 \longrightarrow \mathbb{C}^3, \quad x\begin{pmatrix} -1 \\ -i \\ 1 \end{pmatrix} + y\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + z\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \longmapsto \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

$$= \left\langle \begin{pmatrix} -1 \\ -i \\ 1 \end{pmatrix} \right\rangle \subset U_2$$

$$F_{\mathcal{B}_{U_2}} : U_2 \longrightarrow \mathbb{C}^2, \quad x\begin{pmatrix} -1 \\ -i \\ 1 \end{pmatrix} + y\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \longmapsto \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\mathcal{B}_{U_1} = \left\{ \begin{pmatrix} -1 \\ -i \\ 1 \end{pmatrix} \right\} \rightsquigarrow \mathcal{B}_{U_2} = \left\{ \begin{pmatrix} -1 \\ -i \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$$

$$\rightsquigarrow \mathcal{B}_{U_3} = \left\{ \begin{pmatrix} -1 \\ -i \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

Es5:

$$W = \left\langle \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\rangle$$

$$U_0 = \left\langle \begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\rangle \qquad W_0 = \left\langle \underbrace{\begin{pmatrix} -1 \\ -1 \\ -3 \\ 1 \end{pmatrix}}_{\|}, \begin{pmatrix} 2 \\ 1 \\ 2 \\ 0 \end{pmatrix} \right\rangle$$

$$-2 \begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$U, W \subset V$ s.sp. vettoriali.

$U+W = \{u+w \mid u \in U, w \in W\} \subset V$ s.sp. vettoriali.

$U \cap W \neq \{0_V\} \Rightarrow U+W$

$U \cap W = \{0_W\} \Rightarrow U+W =: U \oplus W$

$$F_{\mathcal{B}} : V \longrightarrow \mathbb{K}^n$$

$$F_{\mathcal{B}_1} : U_1 \longrightarrow \mathbb{C} \quad ; \quad F_{\mathcal{B}_2} : U_2 \longrightarrow \mathbb{C}^2$$

$$x \begin{pmatrix} -1 \\ -i \\ 1 \end{pmatrix} \longmapsto x \qquad\qquad x \begin{pmatrix} -1 \\ -i \\ 1 \end{pmatrix} + y \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \longmapsto \begin{pmatrix} x \\ y \end{pmatrix}$$