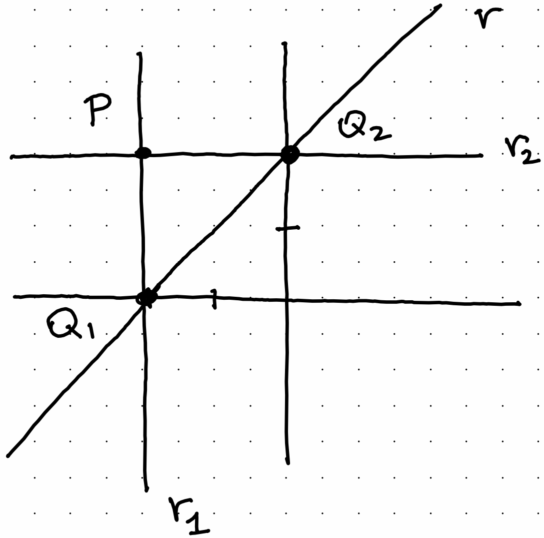
## Es 1 :




$r : x - y = -2$

$r_1 : x = -2 = \begin{pmatrix} -2 \\ 0 \end{pmatrix} + \left\langle \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\rangle$

$r_2 : y = 2 = \begin{pmatrix} 0 \\ 2 \end{pmatrix} + \left\langle \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\rangle$

$Q_1 = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$

$Q_2 = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

Area $\overset{\triangle}{Q_1PQ_2} = 2$

Perimetro : $2 + 2 + \sqrt{8}$

## Es2:

1. $$\det\begin{pmatrix}1&2&3\\-1&-1&2\\2&1&2\end{pmatrix} = \det\begin{pmatrix}1&1&3\\-1&0&2\\2&-1&2\end{pmatrix} = \det\begin{pmatrix}1&1&3\\-1&0&2\\3&0&5\end{pmatrix}$$

$$= -\det\begin{pmatrix}-1&2\\3&5\end{pmatrix} = -(-5-6) = 11 > 0$$

Vol $= 11$.

$\{v_1, v_2, v_3\}$ è una base equiversa di $(\mathbb{R}^3, \cdot)$.

2. $r_1 = X_0 + \langle v_1 \rangle$, $X_0 = \begin{pmatrix}1\\2\\-1\end{pmatrix}$, $v_1 = \begin{pmatrix}2\\-1\\3\end{pmatrix}$ $\qquad r_2 : AX = b \quad : \begin{cases} 2x+y-z=2 \\ x+y+3z=3 \end{cases}$

$Av_1 = \begin{pmatrix}0\\10\end{pmatrix}$, $\quad b - AX_0 = \begin{pmatrix}-3\\3\end{pmatrix}$

$\mathrm{rg}\,(Av_1 \mid b - AX_0) = \mathrm{rg}\begin{pmatrix}0&-3\\10&3\end{pmatrix} = 2 \Rightarrow$ $r_1$ e $r_2$ sono sghembe.

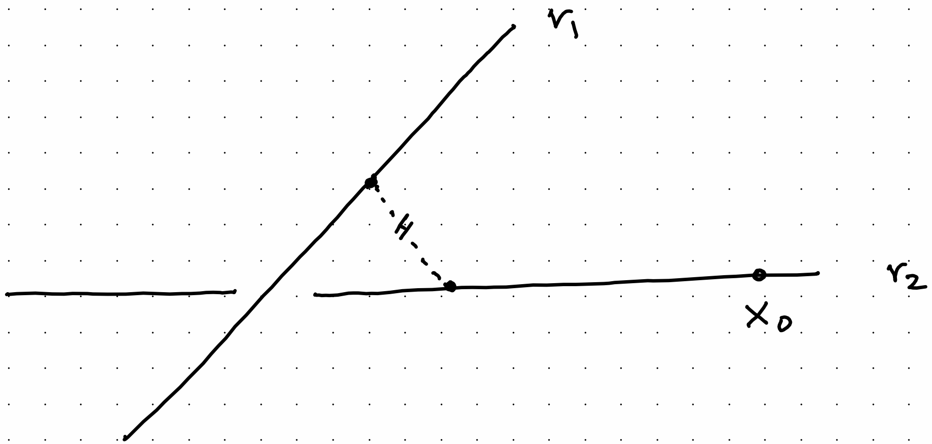
$\pi_{\alpha,\beta}$: $\alpha(2x+y-z-2)+\beta(x+y+3z-3)=0$ piani per $r_2$

$v_1$ sol. di $\alpha(2x+y-z)+\beta(x+y+3z)=0$

$$\alpha(4-1-3)+\beta(2-1+9)=0 \iff 10\beta = 0 \iff \beta = 0$$

$\pi = \pi_{(1,0)}$: $2x+y-z=2$.

$$\mathrm{dist}(r_1,r_2) = \mathrm{dist}(X_0, \pi) = \frac{|2+2+1-2|}{\sqrt{4+1+1}} = \frac{3}{\sqrt{6}}$$




$$\text{dist}(r_1, r_2) \neq \text{dist}(x_0, r_1)$$

Es3:

I) 1)

$$\begin{pmatrix}1\\-1\\2\end{pmatrix} \wedge \begin{pmatrix}-1\\1\\1\end{pmatrix} = \begin{pmatrix}-3\\-3\\0\end{pmatrix}$$

$$\left\| \begin{pmatrix}1\\-1\\2\end{pmatrix} \wedge \begin{pmatrix}-1\\1\\1\end{pmatrix} \right\| = \sqrt{(-3)^2+(-3)^2+0^2} = \sqrt{18}$$

$$\cos \begin{pmatrix}1\\-1\\2\end{pmatrix} \begin{pmatrix}-1\\1\\1\end{pmatrix} = \frac{\begin{pmatrix}1\\-1\\2\end{pmatrix} \cdot \begin{pmatrix}-1\\1\\1\end{pmatrix}}{\left\|\begin{pmatrix}1\\-1\\2\end{pmatrix}\right\| \left\|\begin{pmatrix}-1\\1\\1\end{pmatrix}\right\|} = 0$$

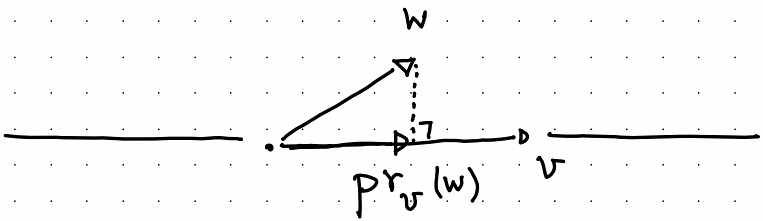
$$v \wedge w = \det(v|w|e_1)\,e_1 + \det(v|w|e_2)\,e_2 + \det(v|w|e_3)\,e_3$$

2) $$\cos \begin{pmatrix}2\\-2\\3\end{pmatrix} \begin{pmatrix}1\\1\\2\end{pmatrix} = \frac{\begin{pmatrix}2\\-2\\3\end{pmatrix} \cdot \begin{pmatrix}1\\1\\2\end{pmatrix}}{\left\|\begin{pmatrix}2\\-2\\3\end{pmatrix}\right\| \left\|\begin{pmatrix}1\\1\\2\end{pmatrix}\right\|} = \frac{6}{\sqrt{17}\sqrt{6}}$$

II) $\boxed{u \cdot v \wedge w = \det(u|v|w)}$ , $(v \wedge w)^t u$

III):

$$\mathrm{pr}_v(w) = \mathrm{pr}_{\langle v\rangle}(w) = \frac{w\cdot v}{v\cdot v}\, v$$




$$\|\mathrm{pr}_v(w)\| = \left\| \frac{w\cdot v}{v\cdot v}\, v \right\| = \frac{|w\cdot v|}{v\cdot v}\, \|v\|$$

$$= \frac{|w\cdot v|}{\|v\|^{\cancel{2}}}\, \cancel{\|v\|} = \frac{|w\cdot v|}{\|v\|}$$

$$\|v\| = \sqrt{v\cdot v}$$

Es 4 :

$$U: \begin{cases} x_1 - x_2 - x_3 + x_4 = 0 \\ 2x_1 - 2x_2 + x_3 + x_4 = 0 \end{cases} \subset \mathbb{R}^4$$

Sol. :

$$U = \operatorname{Ker} \begin{pmatrix} 1 & -1 & -1 & 1 \\ 2 & -2 & 1 & 1 \end{pmatrix} = \operatorname{Ker} \begin{pmatrix} 1 & -1 & 0 & 2/3 \\ 0 & 0 & 1 & -1/3 \end{pmatrix} = \left\langle \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 1 \\ 3 \end{pmatrix} \right\rangle$$

$$A = \begin{pmatrix} 1 & -2 \\ 1 & 0 \\ 0 & 1 \\ 0 & 3 \end{pmatrix} \qquad P_U = A\,(A^t A)^{-1} A^t = \frac{1}{12} \begin{pmatrix} 7 & 5 & -1 & -3 \\ 5 & 7 & 1 & 3 \\ -1 & 1 & 1 & 3 \\ -3 & 3 & 3 & 9 \end{pmatrix}$$

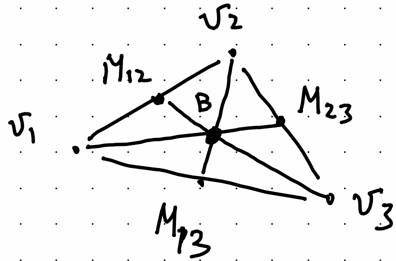
$$pr_U(Q) = P_U Q = \begin{pmatrix} 4 \\ -4 \\ -4 \\ -12 \end{pmatrix}$$

$$\operatorname{dist}(Q, U) = \| Q - pr_U(Q) \| = \| Q - P_U U \| = \left\| \begin{pmatrix} 8 \\ -8 \\ 16 \\ 0 \end{pmatrix} \right\| =$$

$$= 8 \left\| \begin{pmatrix} 1 \\ -1 \\ 2 \\ 0 \end{pmatrix} \right\| = 8\sqrt{6}$$

Es 5 :

1)

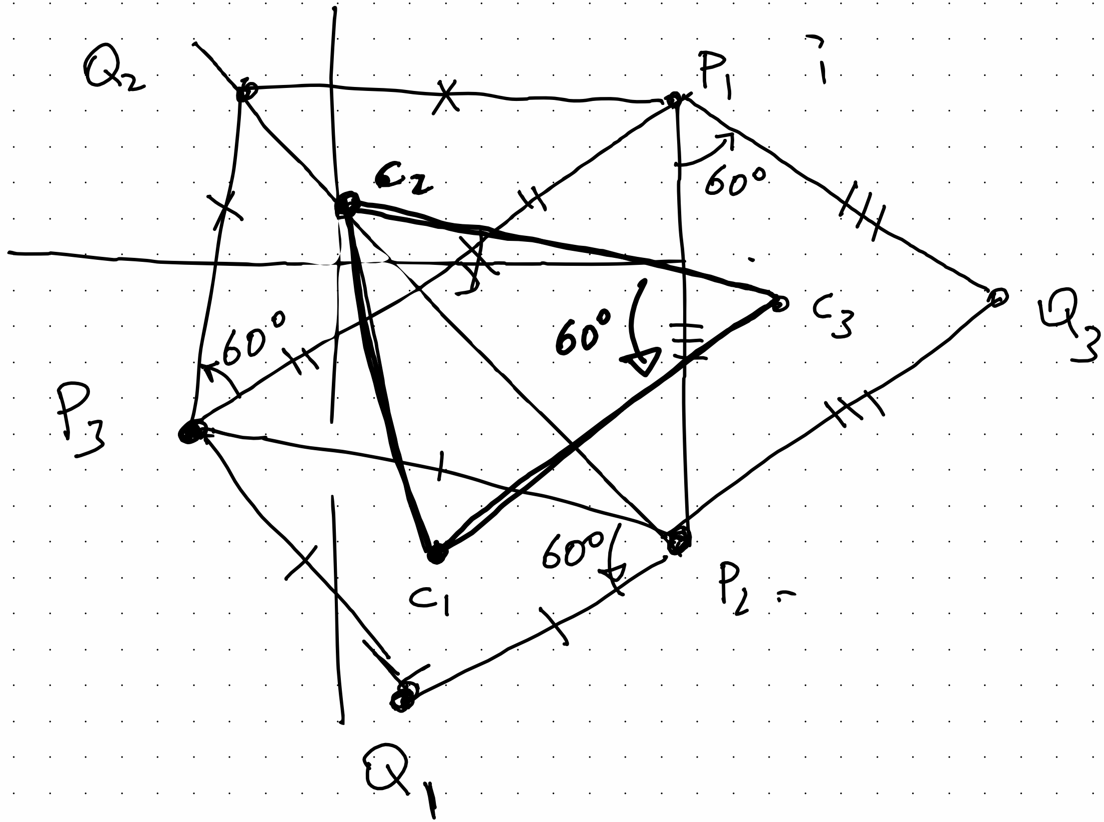



$$M_{12} = \frac{1}{2} v_1 + \frac{1}{2} v_2$$
$$M_{23} = \frac{1}{2} v_2 + \frac{1}{2} v_3$$
$$M_{13} = \frac{1}{2} v_1 + \frac{1}{2} v_3$$

Notiamo che $\frac{1}{3} v_1 + \frac{1}{3} v_2 + \frac{1}{3} v_3 = \frac{2}{3} M_{12} + \frac{1}{3} v_3 =$

$$= \frac{2}{3} M_{23} + \frac{1}{3} v_1 = \frac{2}{3} M_{13} + \frac{1}{3} v_2$$

$\Rightarrow$ $B = \frac{1}{3} v_1 + \frac{1}{3} v_2 + \frac{1}{3} v_3$ appartiene alle tre mediane.



Q2
P1
C2
60°
C3
Q3
60°
60°
P3
60°
P2
C1
Q1

$$P_1 = \begin{pmatrix} 6 \\ 4 \end{pmatrix} \qquad r: \ 2x+y=1 \qquad P_2 = \begin{pmatrix} 6 \\ -11 \end{pmatrix} \in r.$$

$$P_3 = P_2 + Q_{-2}(P_1 - P_2) \qquad Q_m = \frac{1}{1+m^2}\begin{pmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ -11 \end{pmatrix} + \frac{1}{5}\begin{pmatrix} -3 & -4 \\ -4 & 3 \end{pmatrix}\begin{pmatrix} 0 \\ 15 \end{pmatrix} = \begin{pmatrix} -6 \\ -2 \end{pmatrix}$$

$$R = R_{60°} = \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$$

$$Q_1 = P_2 + R(P_3 - P_2) = \begin{pmatrix} -\frac{9}{2}\sqrt{3} \\ -\frac{13}{2} - 6\sqrt{3} \end{pmatrix}$$

$$Q_2 = P_3 + R(P_1 - P_3) = \begin{pmatrix} -3\sqrt{3} \\ 1 + 6\sqrt{3} \end{pmatrix} \in r$$

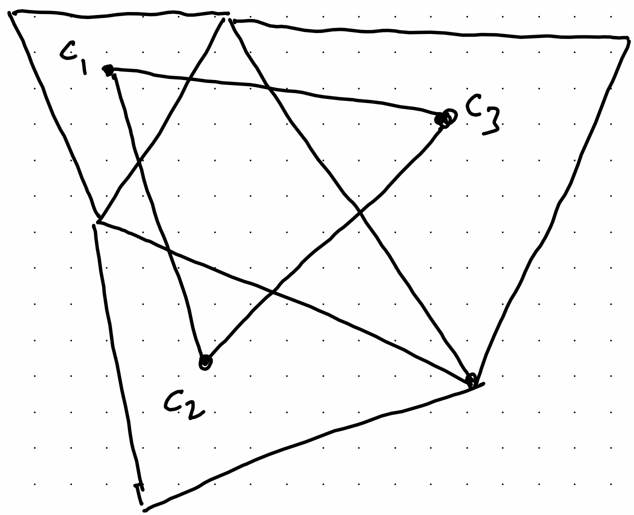
$$Q_3 = P_1 + R(P_2 - P_1) = \begin{pmatrix} 6 + \frac{15}{2}\sqrt{3} \\ -\frac{7}{2} \end{pmatrix}$$

$$c_1 = \frac{1}{3} P_2 + \frac{1}{3} P_3 + \frac{1}{3} Q_1$$

$$c_2 = \frac{1}{3} P_1 + \frac{1}{3} P_3 + \frac{1}{3} Q_2$$

$$c_3 = \frac{1}{3} P_1 + \frac{1}{3} P_2 + \frac{1}{3} Q_3$$

4) Teorema di Napoleone.




$\overset{\triangle}{c_1 c_2 c_3}$ è equilatero.

① Verifichiamo che

$$R(c_2 - c_3) = c_1 - c_3$$

$$R(c_2 - c_3) = \frac{1}{3}\left[ R(P_3 - P_2) - (P_1 - P_3) - (P_1 - P_2) + R(P_1 - P_2) \right]$$

$$c_1 - c_3 = \frac{1}{3}\left[ (P_3 - P_1) + (P_2 - P_1) + R(P_3 - P_2) + R(P_1 - P_2) \right]$$

Si usa questa relazione

$$R^2 - R = -\mathbb{1}_2$$

2

$$\|C_1 - C_2\| = \|C_1 - C_3\| = \|C_2 - C_3\|$$